From Six-Quark Operators to $n - \bar{n}$ Transitions

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Outline

- Theoretical motivations for $n - \bar{n}$ oscillations
- General formalism
- Operator analysis and estimate of matrix elements
- Calculation of $n - \bar{n}$ oscillations in an extra-dimensional model
- Conclusions: This model provides an example of how $n - \bar{n}$ oscillations can occur at rates comparable to current limits, providing motivation for more sensitive experimental searches.

Theoretical Motivations

- Producing the observed baryon asymmetry in the universe requires interactions that violate baryon number, B (Sakharov, 1967).
- General phenomenological possibility of baryon number violation via the $|\Delta B| = 2$ process $n \leftrightarrow \bar{n}$ (Kuzmin, 1970).
- Since (anti)quarks and (anti)leptons are placed in same representations in grand unified theories (GUT's), the violation of B and L is natural in these theories. Besides proton decay, $n - \bar{n}$ oscillations can occur (Glashow, 1979; Marshak and Mohapatra; Chang and Chang, 1980). GUT's are appealing since they (i) embed the SU(3)$_c$, SU(2)$_L$, and U(1)$_Y$ factor groups of the SM in a simple group and hence predict the observed gauge couplings g_3, g_2, and g_Y; (ii) quantize charge, and (iii) unify quarks and leptons.
- Some sources of recent interest in $n - \bar{n}$ oscillations:
  - in some supersymmetric models (Babu and Mohapatra, 2001; Babu, Mohapatra, Nasri, 2006,2007)
  - in models with extra dimension(s) (Nussinov and Shrock, 2002)

General Formalism

$n - \bar{n}$ Oscillations in Field-Free Vacuum:

$\langle n|H_{eff}|n\rangle \ = m_n - i\lambda/2$ and (assuming CPT), $\langle\bar{n}|H_{eff}|\bar{n}\rangle \ = m_n - i\lambda/2$, where H_{eff} denotes relevant Hamiltonian and $\lambda^{-1} = \tau_n = 0.89 \times 10^3$ sec. H_{eff} may also mediate $n \leftrightarrow \bar{n}$ transitions: $\langle\bar{n}|H_{eff}|n\rangle \equiv \delta m$. Consider the 2×2 matrix

$$\mathcal{M} = \begin{pmatrix} m_n - i\lambda/2 & \delta m \\ \delta m & m_n - i\lambda/2 \end{pmatrix}$$

Diagonalizing $\mathcal{M}$ yields mass eigenstates

$$|n_\pm\rangle = \frac{1}{\sqrt{2}}(|n\rangle \pm |\bar{n}\rangle)$$

with mass eigenvalues $m_\pm = (m_n \pm \delta m) - i\lambda/2$.

So if start with pure $|n\rangle$ state at $t = 0$, then there is a finite probability P for it to be an $|\bar{n}\rangle$ at $t \neq 0$:

$$P(n(t) = \bar{n}) = |\langle\bar{n}|n(t)\rangle|^2 = [\sin^2(t/\tau_{n\bar{n}})]e^{-\lambda t}$$

where $\tau_{n\bar{n}} = 1/|\delta m|$. Current limits give $\tau_{n\bar{n}} \gtrsim 10^8$ sec, so $\tau_{n\bar{n}} >> \tau_n$.

$n-\bar{n}$ Oscillations in a Magnetic Field $\vec{B}$:

- Relevant to analysis of reactor experiments searching for $n-\bar{n}$ oscillations
- n and $\bar{n}$ interact with $\vec{B}$ via magnetic moment $\vec{\mu}_{n,\bar{n}}$, $\mu_n = -\mu_{\bar{n}} = -1.9\mu_N$, where $\mu_N = e/(2m_N) = 3.15 \times 10^{-14}$ MeV-Tesla, so

$$\mathcal{M} = \begin{pmatrix} m_n - \vec{\mu}_n \cdot \vec{B} - i\lambda/2 & \delta m \\ \delta m & m_n + \vec{\mu}_n \cdot \vec{B} - i\lambda/2 \end{pmatrix}$$

Diagonalization yields mass eigenstates

$$|n_1\rangle = \cos\theta\ |n\rangle + \sin\theta\ |\bar{n}\rangle\ , \qquad |n_2\rangle = -\sin\theta\ |n\rangle + \cos\theta\ |\bar{n}\rangle$$

where

$$\tan(2\theta) = -\frac{\delta m}{\vec{\mu}_n \cdot \vec{B}}$$

with eigenvalues

$$m_{1,2} = m_n \pm \sqrt{(\vec{\mu}_n \cdot \vec{B})^2 + (\delta m)^2}\ - i\lambda/2$$

Experimentally, reduce $|\vec{B}| = B$ to $B \sim 10^{-4}$ G $= 10^{-8}$ T, so $|\mu_n|B \simeq 10^{-21}$ MeV. Since $|\delta m| << |\mu_n|B$ from exp., $|\theta| << 1$ and $\Delta E \equiv m_1 - m_2 = 2\sqrt{(\vec{\mu}_n \cdot \vec{B})^2 + (\delta m)^2} \simeq 2|\vec{\mu}_n \cdot \vec{B}|$. The transition probability is

$$P(n(t) = \bar{n}) = \sin^2(2\theta)\ \sin^2[(\Delta E)t/2]\, e^{-\lambda t}$$

In a reactor $n - \bar{n}$ experiment, arrange that n's propagate a time t such that $|\vec{\mu}_n \cdot \vec{B}|t << 1$ (and thus also $t << \tau_n$); then

$$P(n(t) = \bar{n}) \simeq (2\theta)^2 \Big(\frac{\Delta E t}{2}\Big)^2 \simeq \left(\frac{\delta m}{\vec{\mu}_n \cdot \vec{B}}\right)^2 \left(\vec{\mu}_n \cdot \vec{B}\, t\right)^2 = [(\delta m)\, t]^2 = (t/\tau_{n\bar{n}})^2$$

Most sensitive reactor $n - \bar{n}$ exps. done with ILL High Flux Reactor (HFR) at Grenoble (Baldo-Ceolin, Fidecaro,.., 1985-1994), last, $L \sim 70$ m, neutrons cooled to liq. D_2 temp., kinetic energy $E \simeq 2 \times 10^{-3}$ eV, vel. $v \simeq 600$ m/s, $t \simeq 0.11$ sec., set limit

$$\tau_{n\bar{n}} \geq 0.86 \times 10^8 \text{ sec} \quad (90\ \%\ CL)$$

i.e., $|\delta m| = 1/\tau_{n\bar{n}} \leq 0.77 \times 10^{-29}$ MeV. Many years since this last reactor experiment; ideas for new reactor exps. (Kamyshkov et al.)

$n - \bar{n}$ Oscillations in Matter:

For $n - \bar{n}$ oscillations involving a neutron bound in a nucleus, consider

$$\mathcal{M} = \begin{pmatrix} m_{n,eff.} & \delta m \\ \delta m & m_{\bar{n},eff.} \end{pmatrix}$$

with

$$m_{n,eff} = m_n + V_n \ , \quad m_{\bar{n},eff.} = m_n + V_{\bar{n}}$$

where the nuclear potential V_n is real, $V_n = V_{nR}$, but $V_{\bar{n}}$ has an imaginary part representing the $\bar{n}N$ annihilation: $V_{\bar{n}} = V_{\bar{n}R} - iV_{\bar{n}I}$ with
$V_{nR}, \ V_{\bar{n}R}, \ V_{\bar{n}I} \sim O(100)$ MeV.
Mixing is thus suppressed; $\tan(2\theta)$ is determined by $2\delta m/(m_{n,eff.} - m_{\bar{n},eff.})$, and

$$\frac{2\delta m}{|m_{n,eff.} - m_{\bar{n},eff.}|} = \frac{2\delta m}{\sqrt{(V_{nR} - V_{\bar{n}R})^2 + V_{\bar{n}I}^2}} << 1$$

Using the reactor exp. bound on $|\delta m|$, this gives $|\theta| \lesssim 10^{-31}$. This suppression in mixing is compensated for by the large number of nucleons in a nucleon decay detector such as Soudan-2 or SuperKamiokande e.g., $\sim 10^{33}$ n's in SuperK.

Eigenvalues:

$$m_{1,2} = \frac{1}{2}\Big[m_{n,eff.} + m_{\bar{n},eff.} \pm \sqrt{(m_{n,eff.} - m_{\bar{n},eff.})^2 + 4(\delta m)^2}\ \Big]$$

Expanding m_1 for the mostly n mass eigenstate $|n_1\rangle \simeq |n\rangle$,

$$m_1 \simeq m_n + V_n - i\frac{(\delta m)^2\, V_{\bar{n}I}}{(V_{nR} - V_{\bar{n}R})^2 + V_{\bar{n}I}^2}$$

Imaginary part leads to matter instability via $\bar{n}n,\ \bar{n}p \to \pi$'s, with mean multiplicity $\langle n_\pi \rangle \simeq 4 - 5$ and rate

$$\Gamma_m = \frac{1}{\tau_m} = \frac{2(\delta m)^2 |V_{\bar{n}I}|}{(V_{nR} - V_{\bar{n}R})^2 + V_{\bar{n}I}^2}$$

So $\tau_m \propto \tau_{n\bar{n}}^2$. Lower bound on $\tau_{n\bar{n}}$ from $n - \bar{n}$ searches in reactor experiments thus yields a lower bound on τ_m and vice versa; with estimated inputs for V_{nR}, $V_{\bar{n}R}$, and $V_{\bar{n}I}$, $\tau_{n\bar{n}} > 0.86 \times 10^8$ s yields $\tau_m \gtrsim 10^{31}$ yr.

Direct limits on matter instability have been reported by IMB, Kamiokande, Frejus, Soudan-2, and SuperK, in particular,

Soudan-2 limit: $\tau_m > 0.72 \times 10^{32}$ yr (90 % CL; Chung et al., 2002), equiv. to $\tau_{n\bar{n}} \gtrsim 1.3 \times 10^8$ sec.

prelim. SuperK limit: $\tau_m > 1.77 \times 10^{32}$ yr (90 % CL; Ganezer et al., 2007), equiv. to $\tau_{n\bar{n}} \gtrsim 3.2 \times 10^8$ sec.

Operator Analysis and Estimate of Matrix Elements

At the quark level $n \longrightarrow \bar{n}$ is $(udd) \longrightarrow (u^c d^c d^c)$. This is mediated by 6-quark operators $\mathcal{O}_i$, so the effective Hamiltonian is

$$\mathcal{H}_{eff} = \sum_i c_i \mathcal{O}_i$$

For d-dimensional spacetime the dimension of a fermion field ψ in mass units is $d_\psi = (d-1)/2$, so dimension $d_{\mathcal{O}_i} = 6 d_\psi = 3(d-1)$ and $d_{c_i} = d - d_{\mathcal{O}_i} = 3 - 2d$. For $d = 4$, $d_\psi = 3/2$, $d_{\mathcal{O}_i} = 9$, and $d_{c_i} = -5$. If the fundamental physics yielding the $n - \bar{n}$ oscillation is characterized by a mass scale M_X, then expect $c_i \sim a_i M_X^{-5}$ so with $H_{eff} = \int d^3x \mathcal{H}_{eff}$, the transition amplitude is

$$\delta m = \langle \bar{n} | H_{eff} | n \rangle = \frac{1}{M_X^5} \sum_i a_i \langle \bar{n} | \mathcal{O}_i | n \rangle$$

Hence $\delta m \sim a \Lambda_{QCD}^6 / M_X^5$, where a is a generic a_i and $\Lambda_{QCD} \simeq 200$ MeV arises from the matrix element $\langle \bar{n} | \mathcal{O}_i | n \rangle$.

Operators $\mathcal{O}_i$ must be color singlets and, for M_X larger than the electroweak symmetry breaking scale, also $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$-singlets. Relevant operators:

$$\mathcal{O}_1 = [u_R^{\alpha T} C u_R^{\beta}][d_R^{\gamma T} C d_R^{\delta}][d_R^{\rho T} C d_R^{\sigma}](T_s)_{\alpha\beta\gamma\delta\rho\sigma}$$

$$\mathcal{O}_2 = [u_R^{\alpha T} C d_R^{\beta}][u_R^{\gamma T} C d_R^{\delta}][d_R^{\rho T} C d_R^{\sigma}](T_s)_{\alpha\beta\gamma\delta\rho\sigma}$$

$$\mathcal{O}_3 = [Q_L^{i\alpha T} C Q_L^{j\beta}][u_R^{\gamma T} C d_R^{\delta}][d_R^{\rho T} C d_R^{\sigma}]\epsilon_{ij}(T_a)_{\alpha\beta\gamma\delta\rho\sigma}$$

$$\mathcal{O}_4 = [Q_L^{i\alpha T} C Q_L^{j\beta}][Q_L^{k\gamma T} C Q_L^{m\delta}][d_R^{\rho T} C d_R^{\sigma}]\epsilon_{ij}\epsilon_{km}(T_a)_{\alpha\beta\gamma\delta\rho\sigma}$$

where $Q_L = \binom{u}{d}_L$, i, j.. are SU(2)$_L$ indices, and color tensors are

$$(T_s)_{\alpha\beta\gamma\delta\rho\sigma} = \epsilon_{\rho\alpha\gamma}\epsilon_{\sigma\beta\delta} + \epsilon_{\sigma\alpha\gamma}\epsilon_{\rho\beta\delta} + \epsilon_{\rho\beta\gamma}\epsilon_{\sigma\alpha\delta} + \epsilon_{\sigma\beta\gamma}\epsilon_{\rho\alpha\delta}$$

$$(T_a)_{\alpha\beta\gamma\delta\rho\sigma} = \epsilon_{\rho\alpha\beta}\epsilon_{\sigma\gamma\delta} + \epsilon_{\sigma\alpha\beta}\epsilon_{\rho\gamma\delta}$$

$(T_s)_{\alpha\beta\gamma\delta\rho\sigma}$ is symmetric in the indices $(\alpha\beta)$, $(\gamma\delta)$, $(\rho\sigma)$.

$(T_a)_{\alpha\beta\gamma\delta\rho\sigma}$ is antisymmetric in $(\alpha\beta)$ and $(\gamma\delta)$ and symmetric in $(\rho\sigma)$.

A given theory determines the coefficients c_i; then one needs to calculate the matrix elements $\langle \bar{n}|\mathcal{O}_i|n\rangle$ to predict δm and thus the resultant $n-\bar{n}$ rate.

Calculation of these matrix elements $\langle \bar{n}|\mathcal{O}_i|n\rangle$ was performed using the MIT bag model (Rao and Shrock, Phys. Lett. B 116, 239 (1982)). Results involve integrals over sixth-power polynomials of spherical Bessel functions from the quark wavefunctions in the bag model. Results:

$$|\langle \bar{n}|\mathcal{O}_i|n\rangle| \sim O(10^{-4})\ GeV^6 \simeq (200\ MeV)^6 \simeq \Lambda_{QCD}^6$$

It would be worthwhile to go beyond the approximations of the MIT bag model and to calculate these matrix elements in full QCD using lattice gauge theory methods.

$n - \bar{n}$ Oscillations in an Extra-Dimensional Model

Current exp. data fully consistent with 4D Minkowski spacetime, but useful to explore possibility of extra dimensions, both from phenomenological point of view and because main candidate theory for quantum gravity - string theory - involves higher dimensions.

Here we focus on theories where SM fields can propagate in the extra dimensions and the wavefunctions of SM fermions have strong localization (with Gaussian profiles) at various points (branes) in this extra-dimensional space. Effective size of extra dimension(s) is L; $\Lambda_L = L^{-1}$ can be ~ 100 TeV, $<< M_{Pl}$.

Such models are of interest partly because they can provide a mechanism for obtaining a hierarchy in fermion masses and quark mixing (e.g., Arkani-Hamed + Schmaltz; Mirabelli + Schmaltz, 2000). Although these are just toy models, they show how $n - \bar{n}$ oscillations can arise in physics beyond the SM.

In generic models of this type, excessively rapid proton decay can be avoided by arranging that the wavefunction centers of the u and d quarks are separated far from those of the e and μ. However, this does not guarantee adequate suppression of $n - \bar{n}$ oscillations. We have analyzed this (Nussinov and Shrock, Phys. Rev. Lett. 88, 171601 (2002); see also Huber and Shafi, Phys. Lett. 512, 365 (2001)).

Denote usual spacetime coords. as x_ν, $\nu = 0, 1, 2, 3$ and consider ℓ extra compact coordinates, y_λ. Let SM fermion have the form $\Psi(x, y) = \psi(x)\chi(y)$, where $\chi(y)$ has support for $0 \le y_\lambda \le L$.

Use a low-energy effective field theory approach with an ultraviolet cutoff M_* and consider only lowest relevant mode in the Kaluza-Klein (KK) mode decompositions of each Ψ field.

To get hierarchy in 4D fermion mass matrices, have the fermion wavefunctions $\chi(y)$ localized with Gaussian profiles of half-width $\mu^{-1} << L$ at various points in the higher-dimensional space:

$$\chi_f(y) = Ae^{-\mu^2|y-y_f|^2}$$

where $|y_f| = (\sum_{\lambda=1}^{\ell} y_{f,\lambda}^2)^{1/2}$.

Starting from the Lagrangian in the d-dimensional spacetime, one obtains the resultant low-energy effective field theory in 4D by integrating over the extra ℓ dimension(s). The normalization factor $A = (2/\pi)^{\ell/4}\mu^{\ell/2}$ is included so that after this integration the 4D kinetic term $\bar{\psi}(x)i\partial\!\!\!/\psi(x)$ has canonical normalization.

Denote $\xi = \mu/\Lambda_L$; choice $\xi \sim 30$ yields adequate separation of fermions while fitting in interval $[0, L]$. (Localization can be produced in a field-theoretic manner for $\ell = 1$ by coupling fermion to scalar field with a kink, similarly for $\ell = 2$.)

A Yukawa interaction in the d-dimensional space with coefficients of order unity and moderate separation of localized wavefunctions yields a strong hierarchy in the effective low-energy 4D Yukawa interaction because the convolution of two of the fermion Gaussian wavefunctions is another Gaussian,

$$\int d^\ell y \bar{\chi}(y_f)\chi(y_{f'}) \sim \int d^\ell y e^{-\mu^2|y-y_f|^2} e^{-\mu^2|y-y_{f'}|^2} \sim e^{-(1/2)\mu^2|y_f-y_{f'}|^2}$$

Have UV cutoff M_* satisfying $M_* > \mu$ for the validity the low-energy effective field theory analysis. Take $\Lambda_L \sim 100$ TeV for adequate suppression of neutral flavor-changing currents; with $\xi = 30$, this yields $\mu \sim 3 \times 10^3$ TeV.

In d-dimensions, $\mathcal{H}_{eff,4+\ell} = \sum_{i=1}^4 \kappa_i O_i$, where the operators O_i are comprised of the $(4+\ell)$-dimensional quark fields corresponding to those in $\mathcal{O}_i$ as Ψ corresponds to ψ. Here mass dimension of coefficients $d_{\kappa_i} = 3 - 2d = -(5+2\ell)$. Hence we write $\kappa_i = \eta_i / M_X^{5+2\ell}$ and, with no loss of generality, take $\eta_4 = 1$. Scale M_X is plausibly $\sim \Lambda_L$.

Now carry out the integrations over y to get, for each i,

$$c_i \mathcal{O}_i(x) = \kappa_i \int d^\ell y O_i(x, y)$$

Consider case $\ell = 2$. Denoting

$$\rho_c \equiv \frac{4\mu^4}{3\pi^2 M_X^9}$$

we find

$$c_i = \rho_c \eta_i \exp\left[-(4/3)\mu^2 |y_{u_R} - y_{d_R}|^2\right] \ , i = 1, 2$$

$$\begin{aligned} c_3 \ &= \ \rho_c \eta_3 \exp[-(1/6)\mu^2 (2|y_{Q_L} - y_{u_R}|^2 + 6|y_{Q_L} - y_{d_R}|^2 \\ &+ \ 3|y_{u_R} - y_{d_R}|^2)] \end{aligned} \tag{1}$$

$$c_4 = \rho_c \exp\left[-(4/3)\mu^2 |y_{Q_L} - y_{d_R}|^2\right]$$

Use fit to data for $\ell = 2$ (Arkani-Hamed and Schmaltz), which gives

$$|y_{Q_L} - y_{u_R}| = |y_{Q_L} - y_{d_R}| \simeq 5\mu^{-1}$$
$$|y_{u_R} - y_{d_R}| \simeq 7\mu^{-1}$$

Can also include corrections due to Coulombic gauge interactions between fermions (Nussinov and Shrock, Phys. Lett. B 526, 137 (2002)).

We find c_j for $j = 1, 2, 3$ are $<<$ c_4, and hence focus on c_4.

To leading order (neglecting small CKM mixings), $|y_{Q_L} - y_{d_R}|$ is determined by m_d via relation

$$m_d = h_d \frac{v}{\sqrt{2}}$$

with

$$h_d = h_{d,0} \exp[-(1/2)\mu^2 |y_{Q_L} - y_{d_R}|^2]$$

where $h_{d,0}$ is the Yukawa coupling in the $(4+\ell)$-dimensional space, so that

$$\exp\left[-(1/2)\mu^2 |y_{Q_L} - y_{d_R}|^2\right] = \frac{2^{1/2} m_d}{h_{d,0} v}$$

Take $h_{d,0} \sim 1$ and $m_d \simeq 10$ MeV; then contribution to δm from $\mathcal{O}_4$ term is

$$\delta m \simeq c_4 \langle \bar{n} | \mathcal{O}_4 | n \rangle \simeq \left(\frac{4\mu^4}{3\pi^2 M_X^9} \right) \left(\frac{2^{1/2} m_d}{v} \right)^{8/3} \langle \bar{n} | \mathcal{O}_4 | n \rangle$$

From MIT bag model calculation we have

$$\langle \bar{n} | \mathcal{O}_4 | n \rangle \simeq 0.9 \times 10^{-4} \ \mathrm{GeV}^6$$

Requiring that the resultant $|\delta m|$ be less than the experimental limit $\tau_{n\bar{n}} > 3 \times 10^8$ sec, i.e., $|\delta m| < 2 \times 10^{-33}$ GeV, we obtain the bound

$$M_X \gtrsim (50 \text{ TeV}) \left(\frac{\tau_{n\bar{n}}}{3 \times 10^8 \text{ sec}} \right)^{1/9}$$
$$\times \left(\frac{\mu}{3 \times 10^3 \text{ TeV}} \right)^{4/9} \left(\frac{|\langle \bar{n}|O_4|n\rangle|}{0.9 \times 10^{-4} \text{ GeV}^6} \right)^{1/9}$$

Uncertainty in calculation of matrix element $\langle \bar{n}|\mathcal{O}_4|n\rangle$ is relatively unimportant for this bound because of the $1/9$ power.

Hence for relevant values of $M_X \sim 50 - 100$ TeV, $n - \bar{n}$ oscillations might occur at levels that are in accord with the current experiment limit but not too far below this limit.

Conclusions

- $n-\bar{n}$ oscillations are an interesting possible manifestation of baryon number violation, of $|\Delta B| = 2$ type, complementary to proton decay. A discovery of $n-\bar{n}$ oscillations would be of profound significance.

- Useful to search for $n - \bar{n}$ oscillations via reactor experiments and massive deep underground detectors studying neutrino oscillations and searching for proton decay. New opportunities could arise with DUSEL.

- Our calculation in an extra-dimensional model provides an example of how new physics beyond the standard model can produce $n - \bar{n}$ oscillations at rates comparable with current experimental limits, as do 4D models yielding $n - \bar{n}$ oscillations. These give motivation for new experimental searches with increased sensitivity.