PROTON DECAY : THE MISSING LINK

(The Need For A NEXT Generation Detector)

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REFERENCES :

JCP, Review Talk, hep-ph/0606089

Babu, Pati, Wilczek, hep-ph/9812538

Babu, Pati, Tavartkiladze — TO APPEAR

I. Introduction

A) Improved Studies of

① Proton Decay

② ν oscillations

2 indispensable tools to probe nature at the shortest distance $\sim 10^{-30}$ cm.

In a broader sense, these two processes get intimately connected with each other within Grand Unification

In fact both are predictions of a well-motivated class of Grand Unification symmetries.

I will therefore begin by first listing evidence which has built up over the years in favor of Grand Unification, in particular for a certain class of Grand Unification Symmetries.

B) Alt. Routes To Grand Unification

$$G(213) = SU(2)_L \times U(1)_Y \times SU(3)^C$$

$$\begin{pmatrix} u_r & u_y & u_b \\ d_r & d_y & d_b \end{pmatrix}_L^{1/3}, \; (u_{r,y,b})_R^{4/3}, \; (d_{r,y,b})_R^{-2/3}, \; \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L^{-1}, \; (e_R^-)^{-2}$$

5 disconn. multiplets // Arb. Y_W, $SU(2)$, $SU(3)^C$ Q. NOS.

$\downarrow$

$$G(224) = SU(2)_L \times SU(2)_R \times SU(4)^C_{L+R} \times (L \leftrightarrow R)$$

$$F^e_{L,R} = \begin{bmatrix} u_r & u_y & u_b & \nu_e \\ d_r & d_y & d_b & e^- \end{bmatrix}_{L,R} \qquad F^e_L = (2,1,4) \quad F^e_R = (1,2,4)$$

$$Q_{em} = I_{3L} + I_{3R} + \frac{B-L}{2}$$

Advantages of G(224)

- All 16 in one L-R Conj. multiplet // L-R symm
- Explain Y_W & All Q. Nos. // Quantize Q_{em} //
- $Q_{e^-} = -Q_p$ // ν_R // B-L $\leftarrow$ Need For Seesaw & Leptogenesis

$\downarrow$

All Advantages of G(224) retained by SO(10), not SU(5)

SO(10) : 16 (one coupling)

SU(5): $\bar{5} + 10$

NO ν_R, NO B-L $\}$ $\rightarrow$ Problem with ν Masses & Leptogenesis.

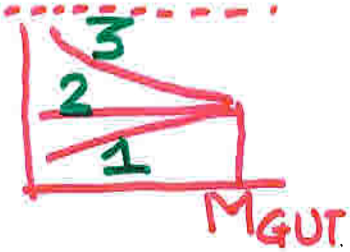
C) EVIDENCE FOR GRAND UNIFICATION

1) Family Structure - Quantum Nos.

2) Charge Quantization

3) $Q_{e^-} = -Q_p$

4) Meeting of The 3 gauge Couplings
   $\downarrow$
   SUSY Grand Unification
   $\downarrow$
   $\boxed{M_{GUT} \simeq 2 \times 10^{16}\ \text{GeV}}$



$\boxed{5)\ \sqrt{\Delta m^2(\nu)_{23}} \approx 1/20\ \text{eV}}$ $\leftrightarrow$ $SU(4)^c \rightarrow \nu_R$ // B-L
$m(\nu^\tau)_{Dirac}$ //
$M_{GUT} \rightarrow$ Maj ν_R
Seesaw

6) $m_b(GUT) = m_\tau$

7) $m(\nu_\tau)_{Dirac} = m_{top}(GUT)$

$\rightarrow SU(4)^c$ — SO(10) // G(224)

8) $\theta(\nu_\mu - \nu_\tau) \approx \pi/4 \longleftrightarrow V_{cb} \simeq 0.04$

9) LEPTOGENESIS $\rightarrow Y_B \sim 10^{-10} \rightarrow \nu_R$/B-L

SUCCESS of All 9 Features Non-Trivial!

TOGETHER MAKE A STRONG CASE FOR SUSY GRAND UNIF // SIMULT. FOR AN EFF. SYMMETRY LIKE SO(10) OR MINIMALLY A STRING-DERIVED G(224) SYMMETRY

II) SO(10) BREAKING / FERMION MASSES / ν OSC

Babu, Pati, Wilczek

$$SO(10) \xrightarrow[\text{LOWEST DIM. HIGGSES}]{\langle 45\rangle,\ \langle 16_H\rangle = \langle \overline{16}_H\rangle} G(213) \xrightarrow{\langle 10_H\rangle} U(1) \times SU(3)^C$$

(NO ~~126~~, ~~120~~, ~~210~~ ..)

YUKAWA:
$$\left\{\begin{array}{l} 16_i\ 16_j\ 10_H \\ 16_i\ 16_j\ 10_H \cdot 45_H/M \\ 16_i\ 16_j\ 16_H\ 16_H/M \end{array}\right\}$$

HIERARCHICAL BY U(1) FLAVOR SYMM $(S/M)^{n_{ij}}$

$\Downarrow$

"33" $\gg$ "23" $\sim$ "32" $\gg$ "22" $\gg$ "12" ETC.

$16_i\ 16_j\ \overline{16}_H\ \overline{16}_H/M \rightarrow$ MAJ. MASS (ν_R)

PATTERN DET. BY SO(10)//G(224) AND Flavor sym.

$\Downarrow$

① A PREDICTIVE FRAMEWORK FOR FERMION MASSES // ν OSC. } BPW

$\Updownarrow$

② CP & FLAVOR VIOLS (B-$\bar{B}$, ϵ'/ϵ, edm, $\mu \to e\gamma$) } Babu, Pati, Rastogi

③ LEPTOGENESIS : $\boxed{Y_B/\sin 2\phi \sim (10\text{-}100) \times 10^{-10}}$ JCP

$\Downarrow$

④ A PREDICTIVE FRAMEWORK FOR PROTON DECAY

$d = 6 \rightarrow p \rightarrow e^+ \pi^0$

$d = 5 \rightarrow p \rightarrow \bar{\nu} K^+$

Summary on Fermion Masses & Mixings (Babu, Pati, Wilczek)

Predictions	Observations
$m_b(m_b) \approx (4.7-4.9)\,\mathrm{GeV}$	$\approx 4.2\,\mathrm{GeV}$
$m(\nu_\tau) \sim (\frac{1}{24}\mathrm{eV})(\frac{1}{2}-2)$	$\approx (\frac{1}{15}-\frac{1}{25})\,\mathrm{eV}$ (*)
$V_{cb} \approx 0.043$	≈ 0.04
$\sin^2 2\Theta^{osc}_{\nu_\mu\nu_\tau} \approx \underbrace{\boxed{0.92}}_{\text{SMA}} \leftrightarrow \underbrace{\boxed{0.99}}_{\text{LMA}}$	$\approx 0.92 \leftrightarrow 1$
$V_{us} \approx 0.21$	≈ 0.22
$\lvert V_{ub}\rvert \approx 0.0032$	$\approx 0.003-0.004$
$m_d(1\,\mathrm{GeV}) \approx 8\,\mathrm{MeV}$	$\approx 8-10\,\mathrm{MeV}$

$m(\nu_\mu) \approx (2-10)\times 10^{-3}\,\mathrm{eV} \longleftrightarrow \begin{cases} \text{SMA} \sim 3\times 10^{-3}\,\mathrm{eV} \\ \boxed{\text{LMA} \approx 7\times 10^{-3}\,\mathrm{eV}} \end{cases}$

$m(\nu_e) \sim (1 \text{ to few})\times 10^{-3}\,\mathrm{eV}$

Consistent with the framework

$M(\text{“}\nu_R^\tau\text{”}, \text{“}\nu_R^\mu\text{”}, \boxed{\text{“}\nu_R^e\text{”}}) \approx (10^{15}, 2\times 10^{12}, (\frac{1}{3}-3)\times 10^{10})\,\mathrm{GeV}$

Just right for Lepto/Baryo genesis

NOTE: ν_R Masses Necessarily Hierarchical.

Summary on Fermion Masses & Mixings in the G(224)/SO(10) Framework

Given the bizarre pattern of masses & mixings of quarks, charged leptons and neutrinos, it seems remarkable that the simple pattern of fermion mass matrices*, motivated in large part by the group th of G(224)/SO(10) and the assumption of minimality of the Higgs system makes 7 predictions in agreement with observation.

→ Study Proton Decay // Leptogenesis // $\not{CP}$ within this framework.

* Need to understand the origin of flavor symmetries.
→ Hierarchical entries.

III PROTON DECAY : HALLMARK OF GRAND UNIFICATION

① $d = 6$ Gauge Med. SUSY SU(5)/SO(10)

$M_X \simeq M_Y \simeq 10^{16}$ GeV $(1 \pm 25\%)$

$\Rightarrow$ Amp $(p \to e^+ \pi^0) \propto g^2/M_X^2$

Chiral Lag Param $(D+F) \simeq 1.25$ // $A_R \simeq 3.4$

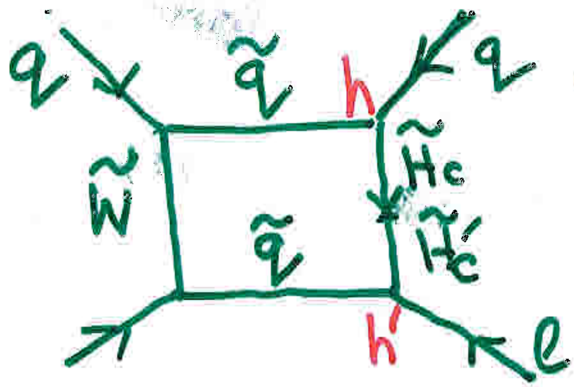
$\alpha_H = 0.01\,\text{GeV}^3$ $(1/\sqrt{2} - \sqrt{2})$ $\Gamma^{-1}(p \to e^+ \pi^0) \simeq 10^{35 \pm 1}$ yrs (Theory)

Can be as short as $\sim 10^{34}$ yrs.

② SUSY SU(5) / SO(10) Standard $d=5$

Color Triplet Higgsino Med Sakai, Yanagida, Weinberg

Need D-T Splitting



$\tilde{H}_c \subset (10_H)_{SO(10)} = (2,2,1) + (1,1,6)$

$$\text{Amp} \propto (h h'/M_{H_c})\,(m_{\tilde{W}}/m^2_{\tilde{q}})\,\alpha_2$$

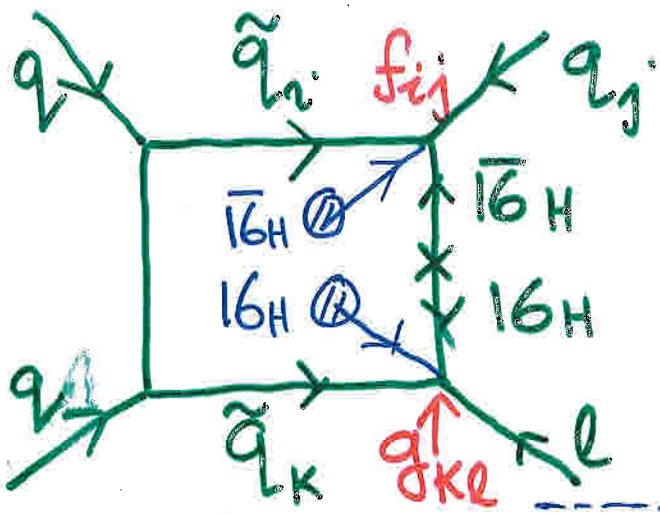
$p \to \bar{\nu} K^+$ (dominant)

$\to \mu^+ K^0$ (suppressed, BR$\sim 10^{-5}$)

$$\boxed{\Gamma^{-1}(p \to \bar{\nu} K^+) \sim 10^{30} - 10^{34} \text{ yrs.}}$$

③ New d=5 ν-Mass Related operators

Babu, Pati, Wilczek



$f_{ij}\, 16_i\, 16_j\, \overline{16}_H \overline{16}_H / M$

⇓

Maj Masses of ν_R's

Generically $16_i\, \overline{16}_H$ in 45 & 1 of SO(10)

$g_{K\ell}\, 16_K\, 16_\ell\, 16_H\, 16_H / M$

↳ $V_{CKM} \neq \mathbb{1}$

Indep of $\tan\beta$

$\Gamma^{-1}(p \to \bar{\nu} K^+)_{\text{New } d=5} \approx 10^{33} - 10^{34}$ yrs

SUSY SO(10) or G(224)

$BR(p \to \mu^+ K^0)_{\text{New } d=5} \approx (10 - 30)\%$

Note these Contributions from new d=5 would generically be present, even if "Standard" d=5 (with Color triplets $\subset 10_H$) absent, as in SUSY G(224) !

The $\mu^+ K^0$ mode a signature of this mechanism.

B) d = 5 proton decay rate Calculation (*)

Babu, Pati, Wilczek (99) // Dermisek, Raby (2000) // Lucas & Raby // Pati (2001 - ph/0106082) // June 2003 – ph/0305221 // KEK Talk ph/0407220) //

Incorporate Dep. of d = 5 p-decay amp. on

- Masses & Mixings of All Fermions
- Unif. Scale Threshold Effects Incl. D-T splitting ⟷ Natural Coupling Unif.
- Take

① $|\beta_H| \simeq |\alpha_H| = (0.01\,\mathrm{GeV}^3)(1/\sqrt{2} - \sqrt{2})$

② $\tan\beta \gtrsim 3$

③ $A_L \simeq 0.32$ (2 Loop)

④ $A_S \simeq 0.93$

⑤ $m_{\tilde{q}} \simeq 1.2\,\mathrm{TeV}\,(1/\sqrt{2} - 2)$ (Focus Point)

⑥ $(m_{\tilde{W}}/m_{\tilde{q}}) \simeq 1/6\,(1/2 - 2)$

Lattice // Domain Wall Fermions + Non-Pert Ren
Quenching error Small
$|\alpha_H| \approx |\beta_H| \simeq 0.01\,\mathrm{GeV}^3$
Aoki et al (2006)

(*) Dimopoulos, Raby, Wilczek (82) // Ellis, Nanopoulos, Rudaz // Nath, Chamseddine, Arnowitt // Nath, Arnowitt (97) // Hisano, Murayama, Yanagida (93) // Hisano (2004) // Babu, Barr (95) //

c) Doublet-Triplet Splitting in SO(10) & M_{eff}

Dimopoulos, Wilczek // Babu, Barr
Barr, Raby // Babu, Pati, Wilczek

$$W_H = \lambda\, 10_H\, 45_H\, 10'_H + M_{10}\, 10'^2_H + M_{16}\, 16_H\, \overline{16}_H$$
$$+ \lambda'\, \overline{16}_H\, \overline{16}_H\, 10_H$$

$$\langle 45_H \rangle = (a, a, a, 0, 0) \times \tau_2 \quad ; \quad a \sim M_{GUT}$$

$\langle 45_H \rangle \propto B-L . \to \langle (1,1,15) \rangle = (a\; a\; a\; {-3a})$.

$$(\bar{5}_{10_H} \;\; \bar{5}_{10'_H} \;\; \bar{5}_{16_H}) \begin{bmatrix} 0 & \lambda\langle 45_H\rangle & \lambda'\langle \overline{16}_H \rangle \\ -\lambda\langle 45_H \rangle & M_{10} & 0 \\ 0 & 0 & M_{16} \end{bmatrix} \begin{pmatrix} 5_{10_H} \\ 5_{10'_H} \\ 5_{\overline{16}_H} \end{pmatrix}$$

$\Rightarrow$ One pair of Higgs doublets light, triplets Heavy

$$H_u = 10_u \; ; \quad H_d = \cos\gamma \; 10_d + \sin\gamma \; 16_d$$

(a) $\tan\gamma = \lambda' \langle \overline{16}_H \rangle / M_{16} \Rightarrow \boxed{\tan\beta = (\cos\gamma)(m_t/m_b)}$

(b) $$\boxed{A(d=5)_{Std} \approx (h_{33}^2/M_{eff})\,(2\times 10^{-5})}$$

$$\boxed{M_{eff} \equiv (\lambda a)^2/M_{10'} \approx M_{GUT}^2/M_{10'}}$$

$\Rightarrow M_{eff} \neq M_{H_C}$; If $M_{10'} < M_{GUT} \Rightarrow \boxed{M_{eff} > M_{GUT}}$

D) STABILIZING D-W // CORELATING M_{EFF} & M_X, & THUS $d=5$ & $d=6$

Babu, Pati, Tavartkiladze (To appear)

Combine BPW (Single $45_H, 16_H, \overline{16}_H, 10_H, 10'_H$)	+	Barr, Raby Add $16' + \overline{16}'$

$\overline{16}_H \cdot 45_H \cdot 16'_H$

$U(1)_{ANOM} \times Z_2 \longrightarrow$ Desired Superpotential

$\rightarrow$ NO UNDESIRABLE PGB

$\rightarrow$ D-W PATTERN STABLE // TO ALL HIGHER ORDER OPERATORS

$\rightarrow \delta\alpha_3(m_Z)$ $-$Ve AS DESIRED $\rightarrow \boxed{\dfrac{m_\Sigma}{m_X} \sim 10^{-2}}$

\+ AN INTERESTING CORELATION $M_{eff} \Leftrightarrow M_X$

$$M_{eff} \simeq 10^{18}\,\mathrm{GeV} \left(\frac{10^{16}\,\mathrm{GeV}}{M_X}\right)^3 \left(\frac{3}{\tan\beta}\right)\left(\frac{1/25}{r}\right) \times$$

$$\times f(p) \exp\left[2\pi \left(\Delta^{(2)}_{2,EW} - \Delta^{(2)}_{3,EW} - \left(\delta\alpha_3^{expt}\right)^{-1}\right)\Big/ 1.4 \times 10^{-2}\right]$$

$$\alpha_3^{-1} = \left(\alpha_3^{(0)}\right)^{-1} - \frac{1}{2\pi}\left[\ln\frac{M_{eff}}{M_X^{(0)}} + 3\ln\frac{\left(M_X^2 M_\Sigma\right)^{1/3}}{M_X^{(0)}} + \ln\frac{16}{9}\,\frac{\tan\beta}{59}\right]$$

$r \equiv m_\Sigma / M_X$

D-T SPLITTING & SUPERPOTENTIAL

Modified Barr, Raby

Babu, JCP, Tavartkiladze

$$\{45_H, 16_H, \overline{16}_H, 10_H, 10'_H\}, \{16', \overline{16}'\}, (S, Z)$$

$A \quad C \quad \bar{C} \quad H \quad H' \quad C' \quad \bar{C}'$ singlets

$$W(A) = M_A A^2 + \frac{\lambda}{M_*} A^4$$

$$W(A, C, C') = C\left[\frac{a_1 ZA}{M_*} + \frac{b_1 C\bar{C}}{M_*} + c_1 S\right]\bar{C}'$$

$$+ C'[a_2 \downarrow \quad b_2 \downarrow \quad c_2 \downarrow]\bar{C}$$

$$W(H, H') = \boxed{HAH'} + \frac{S^K}{M_*^{K-1}}(H')^2 + H\bar{C}\bar{C}$$

	A	H	H'	C	$\bar{C}$	C'	$\bar{C}'$	Z	S
$U(1)_A$	0	1	-1	$\frac{K+4}{2}$	$-\frac{1}{2}$	$\frac{-K+2}{2}$	$\frac{-2K+7}{2}$	$\frac{K+3}{2}$	$\frac{K+3}{2}$
Z_2	$-$	$+$	$-$	$+$	$+$	$+$	$+$	$-$	$+$

$\Rightarrow$ NO operator in any order allowed which could upset DW VEV pattern.

$$\langle A\rangle = i\sigma_2 \otimes \mathrm{Diag}(a, a, a, 0, 0)$$

SUSY SPECTRUM

ASSUME FAMILY UNIVERSAL PARAMETERS
AT HIGH SCALE $M^* \gtrsim M_{GUT}$

Like CMSSM, $\longrightarrow$ MIN. SUGRA
GAUGE / GAUGINO MED
Family UNIV. ANOMALOUS $U(1)$

$$m_0 , \; m_{1/2} , \; \mu , \; \tan\beta$$

Consider A Range

Case 1 $\quad m_0 \simeq 300\,\text{GeV}, \; m_{1/2} \simeq 352\,\text{GeV}$

$\mu = 1\,\text{TeV} \quad , \quad \tan\beta = 3$

Case 2 $\quad m_0 \simeq 930\,\text{GeV}, \; m_{1/2} \simeq 147\,\text{GeV}$

$\mu = 1\,\text{TeV} \quad , \quad \tan\beta = 3$

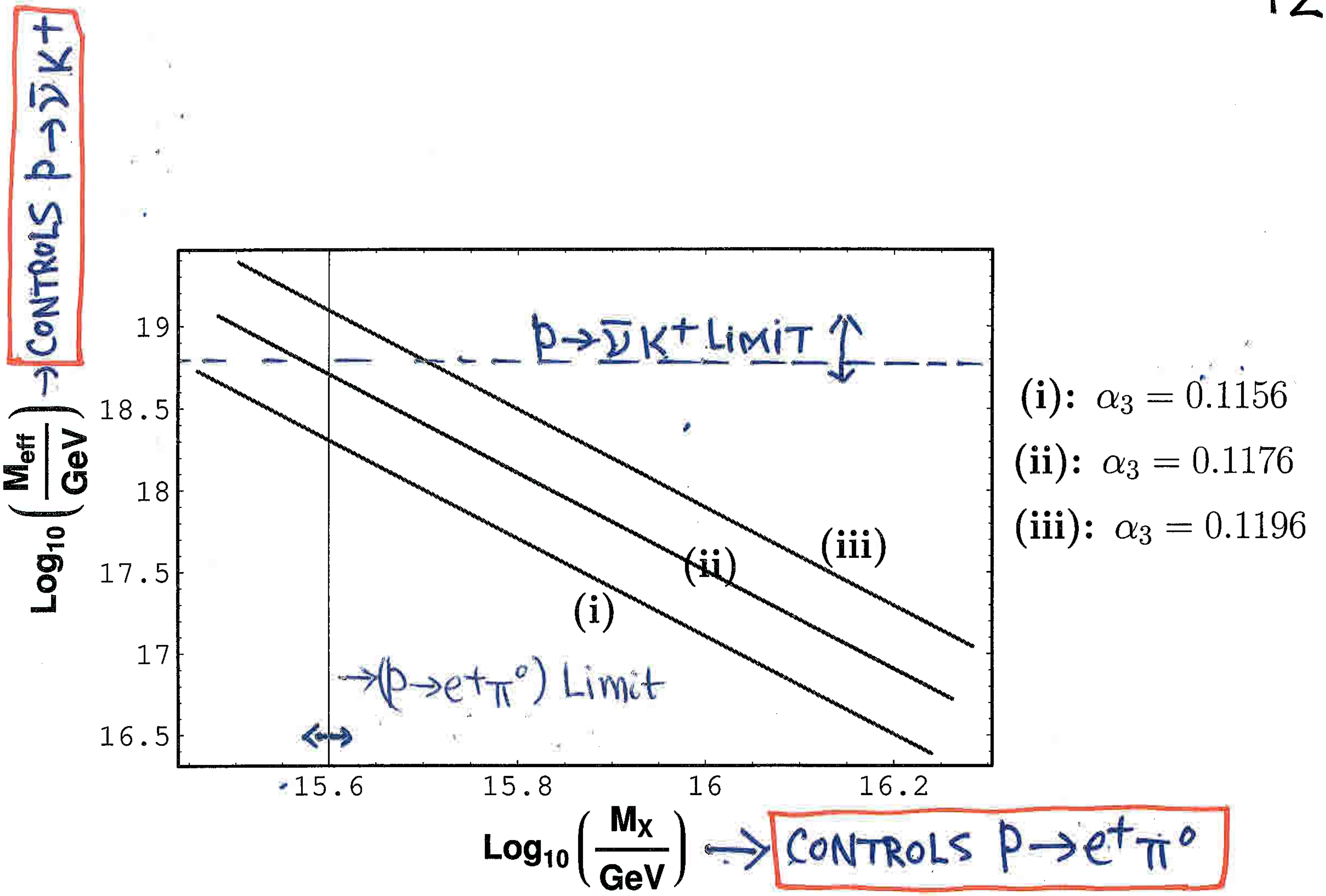


Figure 1: $r = 1/25, \quad \tan\beta = 3.$

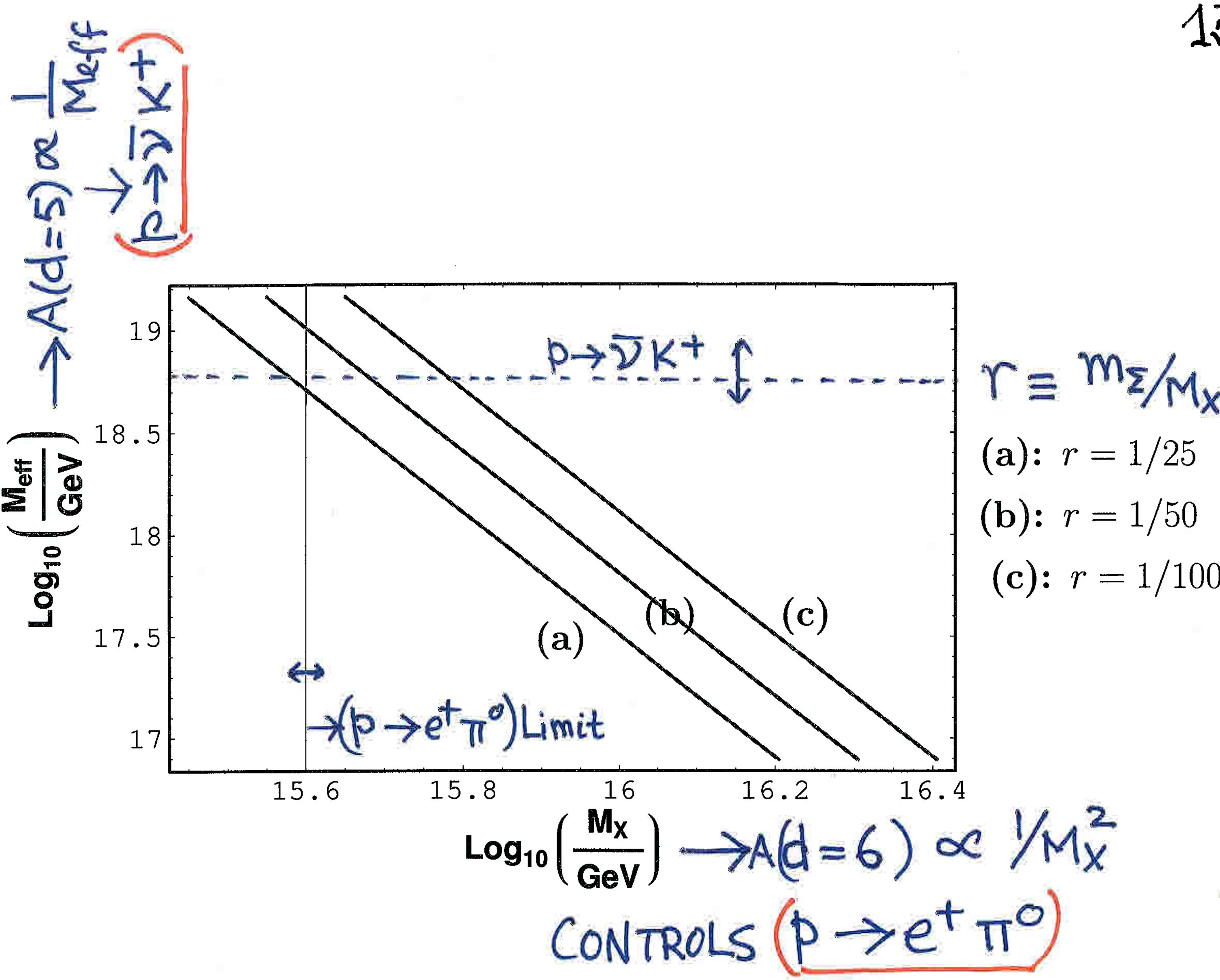


Figure 1: $\alpha_3(M_Z) \simeq 0.1176, \quad \tan\beta = 3.$

$$\langle 45_H \rangle = (a, a, a, 0, 0) \times \tau_2$$

$$\langle 16_H \rangle = c$$

$$\boxed{p \equiv 2c/a} \quad \text{Expect } \boxed{p \sim \mathcal{O}(1)}$$

$$M^2_{X,Y} = g^2_{10}\, a^2 , \quad M^2_{X',Y'} = M^2_X (1+p^2)$$

$$\Gamma(p \to e^+ \pi^0) \simeq \left[\frac{m_p}{64\pi f_\pi^2}\right] (1+D+F)^2 \alpha^2_{Had} \left(\frac{g^2_{GUT} A_R}{M_X^2}\right)^2 \underbrace{\left[5 + \frac{2}{1+p^2} + \frac{1}{(1+p^2)^2}\right]}_{\Phi(p) = 25/4 \text{ For } p=1}$$

SU(5) limit

$$\Gamma^{-1}(p \to e^+ \pi^0) = (0.8 \times 10^{35}\, \text{yrs}) \left(\frac{\alpha_H}{0.01\, \text{GeV}^3}\right)^{-2} \left(\frac{\alpha_{GUT}}{1/23}\right)^{-2} \times \left(\frac{A_R}{3.2}\right)^{-2} \left(\frac{M_X}{10^{16}\, \text{GeV}}\right)^4 \left[\frac{\Phi(p)}{25/4}\right]^{-2}$$

$$\boxed{\Gamma^{-1}(p \to e^+ \pi^0)_{SO(10)} \simeq 0.8 \times 10^{35 \pm 1}\ \text{yrs}}$$

Note $\Phi(p)$ varies from 5 to 8 as $p \to \infty$ to 0

SO(10) d=5

$$\Gamma^{-1}(p\to\bar{\nu}_\tau K^+)$$

$$\simeq (3\times10^{32}\,\text{yrs})\times\left(\frac{.32}{A_L}\right)^2\times\left(\frac{.93}{A_S}\right)^2\times\left(\frac{.01}{\beta_H}\right)^2$$

$$\times\left[(1/6)/m_{\tilde{w}}/m_{\tilde{q}}\right]^2\times\left[\frac{m_{\tilde{q}}}{1.2\,\text{TeV}}\right]^2\times\left[\frac{M_{eff}}{8\times10^{18}\,\text{GeV}}\right]^2$$

Uncertainty $\approx (1/4-4)(1/4-4)(\tfrac{1}{4}-2)(1/2-2)$

$\approx (1/128 - 64)$

$$\Rightarrow \boxed{\Gamma^{-1}(p\to\bar{\nu}K^+)\Big|_{SO(10)} \approx 2\times10^{30} - 2\times10^{34}\ \text{yrs.}}$$

SUMMARY ON PROTON DECAY (d=5)

SUSY Min. SU(5) / CMSSM: $\boxed{\bar{\Gamma}^{-1}(p\to\bar{\nu}K^{+}) < 10^{32}\ \text{yrs}}$ EXCLUDED BY SuperK

SUSY SO(10) / CMSSM: $\left\{ \begin{array}{l} \boxed{\bar{\Gamma}^{-1}(p\to\bar{\nu}K^{+})_{\text{Median}} \lesssim 2\times10^{33}\ \text{yrs}} \\ \bar{\Gamma}^{-1}(p\to\bar{\nu}K^{+})_{\text{Heavy}} < 2\times10^{34}\ \text{yrs} \end{array} \right.$ TIGHTLY CONSTRAINED Std d=5

SUSY SO(10) / CESSM: $\boxed{\begin{array}{l} \bar{\Gamma}^{-1}(p\to\bar{\nu}K^{+})_{\text{Median}} \lesssim 2\times10^{34}\ \text{yrs} \\ \qquad\qquad\qquad\quad \text{HEAVY} \lesssim 2\times10^{35}\ \text{yrs} \end{array}}$ Std. d=5

SUSY G(224)/SO(10) / CMSSM or CESSM: $\boxed{\bar{\Gamma}^{-1}(p\to\bar{\nu}K^{+})_{\text{Median}} \approx 10^{33}-10^{34}\ \text{yrs}}$ NEW ν-Rel d=5

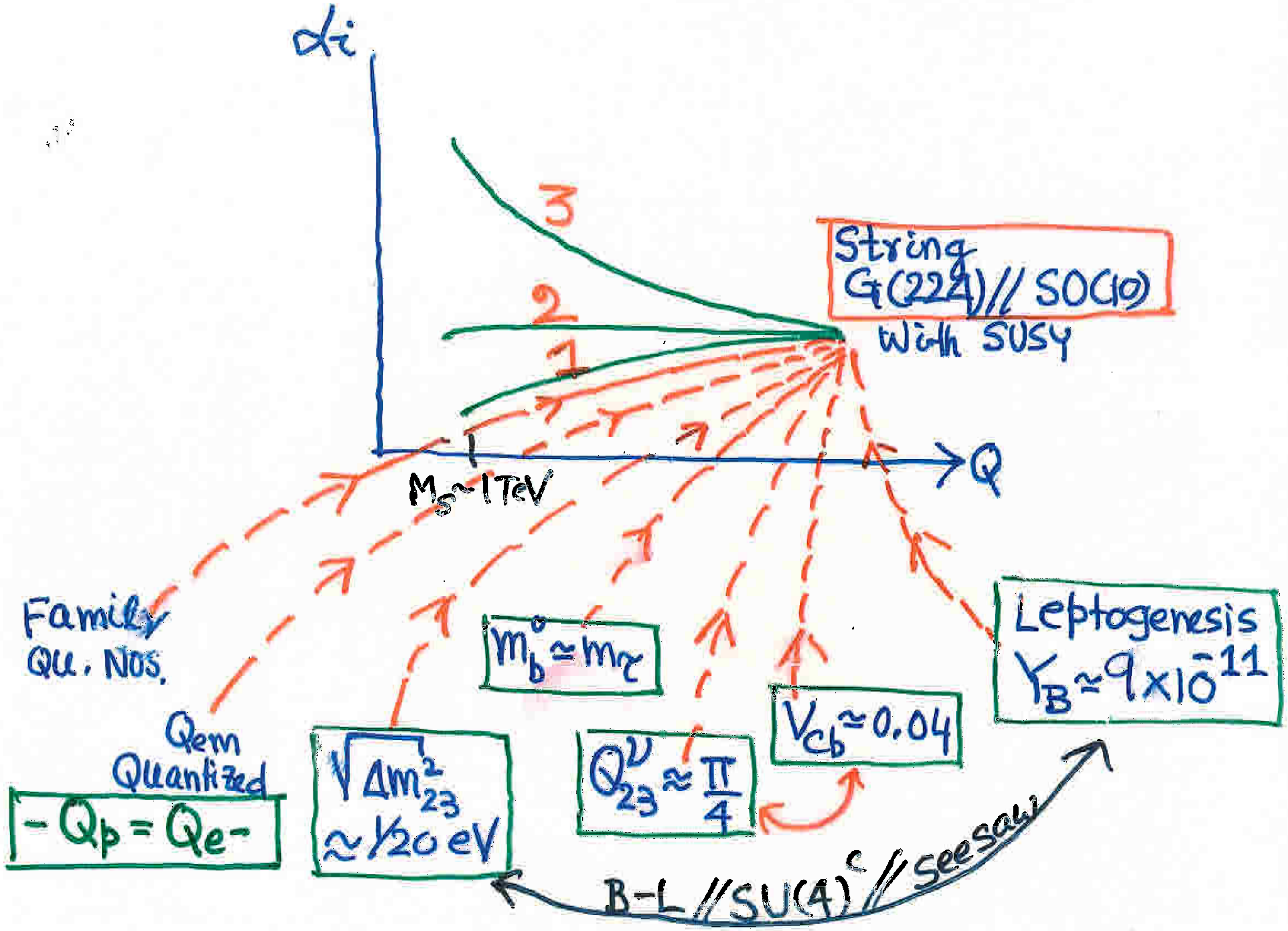
Last 3 Fully Compatible with present limits // MUST BE SEEN WITH IMPROVEMENT BY FACTOR 5-10 // OR ELSE MAJOR SET-BACK

d = 6

SUSY SU(5) $\to \bar{\Gamma}^{-1}(p\to e^{+}\pi^{0}) \simeq 10^{35.3\pm1}$ yrs

SUSY SO(10) $\to \boxed{\bar{\Gamma}^{-1}(p\to e^{+}\pi^{0}) \simeq 0.8\times10^{35\pm1}\ \text{yrs.}}$

VIII Rendevouz In Particle Physics



All these features hang together neatly within a single unified framework → Hard to believe this can be a mere coincidence.

Rendevous Incomplete → Two Missing Links

1) Supersymmetry → LHC

2) Proton Decay → Need Next Gen. Detector.

PROTON DECAY THE HALLMARK OF GRAND UNIF.

Provides a Unique Window To View Physics at truly short dist $\lesssim 10^{-30}$ cm

$p \to \bar{\nu} K^+$ If Seen $\Rightarrow \begin{cases} q\text{-}\ell,\ q\text{-}\bar{q},\ q\text{-}\bar{\ell} \text{ unif } // \\ \text{SUSY UNIF } // \\ M_X \sim 10^{16} \text{ GeV} \end{cases}$

$p \to \mu^+ K^0$ If Seen $\Rightarrow \begin{cases} \nu \text{ MASS Rel op} \\ \text{IMPORTANT} \end{cases}$

$\left.\begin{array}{l} \text{If } e^+\pi^0 \text{ not seen} \\ \text{Say upto } 10^{36} \text{ yrs, but} \\ \bar{\nu}K^+ \text{ seen} \end{array}\right\} \Rightarrow \begin{cases} \text{EFF. } G(224) \ \& \\ \nu \text{ MASS Rel} \\ d=5 \text{ IMPORTANT} \end{cases}$

$\boxed{\text{THUS PROTON DECAY IF SEEN WILL BRING A WEALTH OF KNOWLEDGE THAT CAN NOT BE GAINED BY ANY OTER MEANS}}$

$\Downarrow$

DISCOVERY POTENTIAL HIGH

$\Downarrow$

MEGATON SIZE DETECTOR SENSITIVE TO PROTON DECAY & ν oscillations & ...!

Background Materials

FLAVOR SYMMETRY : $SO(10) \times U(1)_F$

BPW MODEL

Assume Hierarchical Yukawa Couplings

$$\text{"33"} \gg \text{"23"} \sim \text{"32"} \gg \text{"22"} \gg \text{"12" etc.}$$

$\uparrow$

Due to Flavor Symmetry $U(1)_F$

	16_3	16_2	16_1	10_H	16_H	$\overline{16}_H$	45_H	S
$U(1)_F \Rightarrow$	a	$a+1$	$a+2$	$-2a$	$-a-\frac{1}{2}$	$-a$	0	-1

Thus, $\underset{a}{16_3}\ \underset{a}{16_3}\ \underset{-2a}{10_H} \rightarrow$ Allowed ~ 1

$$16_3\, 16_2\, 10_H \langle S/M \rangle \rightarrow \left(\frac{M_{GUT}}{M_{st}} \sim 1/10 \right)$$

$\langle S \rangle \sim M_{GUT}$, $M \sim M_{string}$

A Concrete Example: Minimal Higgs: $\{45_H, 16_H, \overline{16}_H, 10_H\} + \text{"}S\text{"}$

1st consider only μ & τ Families (Flavor sym: $\mu \neq \tau$) (BPW)

$$\mathcal{L}_{mass} = h_{33}\, 16_3\, 16_3 \langle 10_H \rangle \longrightarrow \text{3rd Family: } m_b^0 = m_\tau^0$$

$\propto 1$

$\propto \sigma \quad + \tilde{h}_{23}\, 16_2\, 16_3 \langle 10_H \rangle \langle S/M \rangle \longrightarrow$ 2nd Family

ANTISYMMETRIC, $\propto B-L$

$\propto \epsilon \quad + a_{23}\, 16_2\, 16_3 \dfrac{\langle 10_H \rangle \langle 45_H \rangle}{M} \rightarrow m_\mu^0 \neq m_s^0$

$\propto \hat{\eta} \quad + g_{23}\, 16_2\, 16_3\, \underline{\langle 16_H \rangle}\, \langle 16_H^D \rangle_{EW} / M$

↳ CKM $\neq 1$

$$10_H \rightarrow (2 \leftrightarrow 2) \ll (2 \leftrightarrow 3) \ll (3,3) \text{ Flavor sym}$$

$$U = \begin{array}{c} \begin{array}{cc} c & t \end{array} \\ \begin{pmatrix} 0 & \epsilon + \sigma \\ -\epsilon + \sigma & 1 \end{pmatrix} \end{array} m_U^0 \qquad D = \begin{array}{c} \begin{array}{cc} s & b \end{array} \\ \begin{pmatrix} 0 & \epsilon + \eta \\ -\epsilon + \eta & 1 \end{pmatrix} \end{array} m_D^0$$

$$N_{Dirac} = \begin{array}{c} \begin{array}{cc} \nu_\mu & \nu_\tau \end{array} \\ \begin{pmatrix} 0 & -3\epsilon + \sigma \\ +3\epsilon + \sigma & 1 \end{pmatrix} \end{array} m_U^0 \qquad L = \begin{array}{c} \begin{array}{cc} \mu & \tau \end{array} \\ \begin{pmatrix} 0 & -3\epsilon + \eta \\ 3\epsilon + \eta & 1 \end{pmatrix} \end{array} m_D^0$$

Note $q - \ell$ correlation (SU(4)-color)

up-down " $(SU(2)_L \times SU(2)_R)$

$$\boxed{\eta \equiv \hat{\eta} + \sigma}$$

Dirac Masses (3 Families)

$$U = \begin{pmatrix} 0 & \epsilon' & 0 \\ -\epsilon' & 0 & \epsilon+\sigma \\ 0 & -\epsilon+\sigma & 1 \end{pmatrix} m_U^0 \; ; \; D = \begin{pmatrix} 0 & \epsilon'+\eta' & 0 \\ -\epsilon'+\eta' & 0 & \epsilon+\eta \\ 0 & \epsilon+\eta & 1 \end{pmatrix} m_D^0$$

$$N = \begin{pmatrix} 0 & -3\epsilon' & 0 \\ 3\epsilon' & 0 & -3\epsilon+\sigma \\ 0 & 3\epsilon+\sigma & 1 \end{pmatrix} m_U^0 \; ; \; L = \begin{pmatrix} 0 & \epsilon'+\eta' & 0 \\ -\epsilon'+\eta' & 0 & -3\epsilon+\eta \\ 0 & +3\epsilon+\eta & 1 \end{pmatrix} \times m_D^0$$

---

2 New param (ϵ', η'), but 5 new observables just in (q, ℓ) System $\Rightarrow$ 3 New predictions for (q,ℓ) // With $\epsilon' = 0$ $\rightarrow m_u \rightarrow 0$

ν Majorana Masses

$$M_R^{\nu} = \begin{pmatrix} x & 0 & z \\ 0 & 0 & y \\ z & y & 1 \end{pmatrix} M_R$$

Saw before, $M_R \approx 10^{15}$ GeV

Expect $y \sim 1/10$

$$f_{ij}\, 16_i\, 16_j \langle \overline{16}_H \rangle \langle \overline{16}_H \rangle / M_{st}$$

$$\downarrow$$

$$f_{ij}\, \nu_{iR}^T\, C^{-1} \nu_{jR} \langle \overline{16}_H \rangle^2 / M_{st}$$

$$(M_R^{\nu})_{ij} = f_{ij} \langle \overline{16}_H \rangle^2 / M_{st}$$

Note Same hier. pattern as in Dirac Sector

Including $m_U^0 \to$ 7 param $(\eta, \epsilon, \sigma, \eta', \epsilon', m_U^0, m_D^0)$

describing $9 \times 4 = 36$ entries $\longrightarrow$ Will it work ?

Input: Assume all param real for a moment

$m_t^{phys} = 174$ GeV ; $m_c(m_c) = 1.37$ GeV,
$m_s(1\,\text{GeV}) = 116$ MeV , m_μ , m_τ , $m_u(M_X) = 1.5$ MeV, m_e

$\Downarrow$

$\sigma \simeq 0.110$, $\eta \simeq 0.151$, $\epsilon \simeq -0.095$,
$\epsilon' = \sqrt{m_u/m_c}\,(m_c/m_t) \simeq 2 \times 10^{-4}$; $\eta' \simeq \sqrt{m_e/m_\mu}\left(\frac{m_c}{m_t}\right) \simeq 4 \times 10^{-3}$
$m_U^0 = m_t(M_X) \simeq 110$ GeV ; $m_D^0 \simeq 1.5$ GeV

Majorana Mass of ν_R's : $f_{ij}\, 16_i\, 16_j\, \overline{16}_H\, \overline{16}_H / M$

$\Rightarrow f_{ij}\, (\nu_R^{i\,T} C^{-1} \nu_R^j)\, \langle 16_H \rangle^2 / M$

$$M_R^\nu = \begin{pmatrix} x & 0 & z \\ 0 & 0 & y \\ \cancel{z} & y & 1 \end{pmatrix} \boxed{M_R}$$

determined by $m_{\nu_2}/m_{\nu_3} \approx 1/6$

Calculated $\simeq 5 \times 10^{14}$ GeV.

6 New observables.

SMA or LMA ?

$f_{ij}\, 16_i\, 16_j\, \overline{16}_H\, \overline{16}_H / M$

Just with Standard See-Saw ν_L-masses, SMA rather generic

$m(\nu_L^e) \sim 2\times10^{-5} - 2\times10^{-6}\,\mathrm{eV}$ // $m(\nu_L^\mu) \sim 3\times10^{-3}\,\mathrm{eV}$

$\Theta^{osc}_{\nu_e\nu_\mu} = \Theta^{\ell}_{e\mu} - \Theta^{\nu}_{e\mu} \approx 0.05$

Situation alters once allow for direct Maj. masses of ν_L's — Most likely to arise through Higher Dim. op. involving GUT & EW VEV's - Through tiny $\sim 10^{-3}$ eV entries → IMPORTANT For $(\nu_e - \nu_\mu)$

$$W \supset g_{12}\, \underbrace{16_1\, 16_2}\, 16_H\, 16_H\, \underbrace{10_H\, 10_H} / M^3_{GUT}$$

$$g_{12}\,(\nu_L^e\, \nu_L^\mu)\,\left(\langle \tilde{\nu}_{RH}\rangle / M_{GUT}\right)^2 \cdot (v_u^2)$$

$(\sim 175\,\mathrm{GeV})^2$

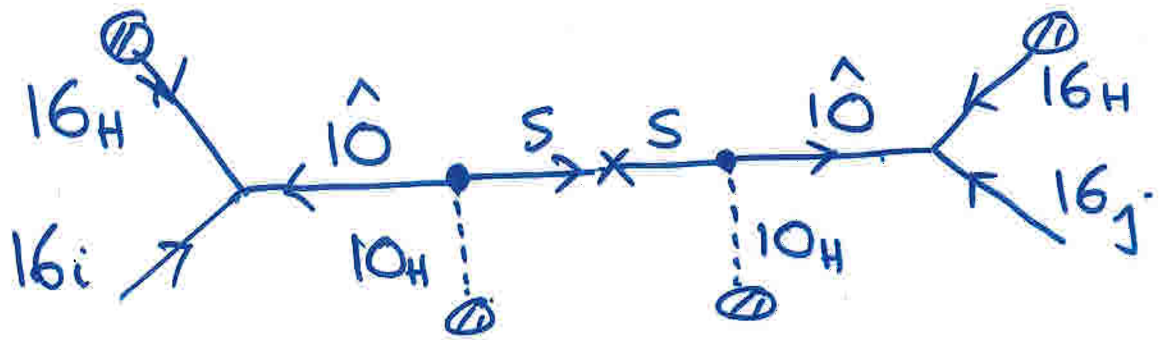
$$\boxed{\sim g_{12}(\nu_L^e\, \nu_L^\mu)\,(1.5-6)\times10^{-3}\,\mathrm{eV}}$$

$(\langle 16_H\rangle \approx (1-2)\, M_{GUT})$

$$\begin{array}{cc} \nu_L^e & \nu_L^\mu \\ \end{array}$$
$$\begin{bmatrix} \approx 0 & (3-4) \\ (3-4) & 7 \end{bmatrix} \times 10^{-3}\,\mathrm{eV} \Longrightarrow \begin{bmatrix} \Theta^{\nu}_{\nu_e\nu_\mu} \approx 1/2 \\ \sin^2 2\Theta^{osc}_{\nu_e\nu_\mu} \sim 0.7 \end{bmatrix}$$ Quite Plausible

Thus LMA not strictly a prediction, but perfectly plausible within the framework.

GUT-SCALE EFFECTIVE OPERATORS



$$\Downarrow$$

$$g_{ij}\ 16_i\, 16_j\, 16_H\, 16_H\ 10_H\, 10_H \big/ (M^2_{\hat{10}} M_S)$$

Θ_{13}

$$m(\nu_L^e \nu_L^\tau)_{\text{Non-seesaw}} \sim (2-6)\times 10^{-3}\,\text{eV}$$

$$\Theta_{13} \sim \frac{(2\text{-}6)\times 10^{-3}\,\text{eV}}{5\times 10^{-2}\,\text{eV}}$$

$$\sim .03 - .1$$

ν-less 2β decay : $\Delta L = \pm 2$

$$m_{ee} = \left|\sum_i m_i U_{ei}^2\right|$$

$m_1 \sim$ few $\times 10^{-3}$ eV, $m_2 \approx (6\text{-}8)\times 10^{-3}$ eV

$m_3 \approx 5\times 10^{-2}$ eV

$\Theta_{12} \approx 1/2$, $\Theta_{13} \sim 0.03 - 0.1$

$$\Downarrow$$

$$m_{ee} \sim (1 \text{ to } 6)\times 10^{-3}\,\text{eV}$$