

Neutrino Masses and Proton Decay- A Realistic SO(10)

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International Workshop: Search for Baryon and Lepton Number Violation

Outline

A Minimal SO(10) Model

Proton decay

Neutrino masses and other predictions

Lepton Flavor Violation

Conclusion

The

The Yukawa superpotential involves the couplings
couplings of 16-dimensional matter spinors

ψ_i (i denotes a generation index)

with 10 (**H**), $\overline{126}$ ($\overline{\Delta}$), and 120 (**D**) dim. Higgs
fields:

$$W_Y = 1/2 h_{ij} \psi_i \psi_j H + 1/2 f_{ij} \psi_i \psi_j \Delta + 1/2 h'_{ij} \psi_i \psi_j D.$$

h and f are symmetric matrices and h' is an anti-symmetric matrix due to the $SO(10)$ symmetry.

The Higgs doublet fields not only exist in **H**, Δ , $\overline{D}$, but also
also exist in other Higgs fields (e.g., **210**) which are needed
needed in the model.

The

210 Higgs multiplet (ϕ): is employed to break the $SO(10)$ symmetry down to the standard model

$\overline{126}$ Higgs multiplet ($\overline{\Delta}$) : introduced as a vector-like pair and this field also contains a Higgs doublet.

The VEV of this pair reduces the rank of $SO(10)$ group, keep supersymmetry unbroken down to the weak scale.

Altogether, we have **six** pairs of Higgs doublets:

$$\phi_d = (H_d^{10}, D_d^1, D_d^2, \Delta_d, \overline{\Delta_d}, \phi_d),$$

$$\phi_u = (H_u^{10}, D_u^1, D_u^2, \Delta_u, \overline{\Delta_u}, \phi_u),$$

where superscripts 1, 2 of $D_{u,d}$ stand for $SU(4)$ singlet and adjoint adjoint pieces under the $G_{422} = SU(4) \times SU(2) \times SU(2)$ decomposition.

The mass term of the Higgs doublets : $(\phi_d)_a (M_D)_{ab} (\phi_u)_b$,

The lightest Higgs pair (MSSM doublets) has masses of the order order of the weak scale.

The

The above Yukawa interaction includes mass terms of the quark and lepton fields as follows:

$$\begin{aligned} W^{\text{mass}}_Y = & h H^{10}_d (q d^c + \ell e^c) + h H^{10}_u (q u^c + \ell v^c) + \\ & 1/\sqrt{3} f \Delta_d (q d^c - 3 \ell e^c) + 1/\sqrt{3} f \Delta_u (q u^c - 3 \ell v^c) + \sqrt{2} f v^c v^c \Delta_R + \\ & \sqrt{2} f \ell \ell \Delta_L + h' D^1_d (q d^c + \ell e^c) + h' D^1_u (q u^c + \ell v^c) + \\ & 1/\sqrt{3} h' D^2_d (q d^c - 3 \ell e^c) - 1/\sqrt{3} h' D^2_u (q u^c - 3 \ell v^c), \end{aligned}$$

where $q, u^c, d^c, \ell, e^c, v^c$ are the quark and lepton fields for the standard model, which are all unified into one spinor representation of $SO(10)$.

Imposing that the Lagrangian is invariant under a CP conjugation, the Yukawa couplings, h_{ij} , f_{ij} and h'_{ij} and all masses and couplings in the Higgs superpotential are all real.

The mixing of the lightest Higgs doublets with the Higgs doublets doublets present in **120** involves a pure imaginary coefficient which will make the fermion masses **hermitian** in this model

The

The Yukawa coupling matrices for fermions:

$$\begin{aligned} Y_u &= \bar{\mathbf{h}} + \mathbf{r}_2 \bar{\mathbf{f}} + \mathbf{r}_3 \bar{\mathbf{h}}', \quad Y_d = \mathbf{r}_1(\bar{\mathbf{h}} + \bar{\mathbf{f}} + \bar{\mathbf{h}}'), \\ Y_e &= \mathbf{r}_1(\bar{\mathbf{h}} - 3\bar{\mathbf{f}} + \mathbf{c}_e \bar{\mathbf{h}}'), \quad Y_\nu = \bar{\mathbf{h}} - 3\mathbf{r}_2 \bar{\mathbf{f}} + \mathbf{c}_\nu \bar{\mathbf{h}}', \end{aligned}$$

where the subscripts **u**, **d**, **e**, **ν** denotes for up-type quark, down-type quark, charged-lepton, and Dirac neutrino Yukawa couplings, respectively,

$$\begin{aligned} \bar{\mathbf{h}} &= V_{11}\mathbf{h}, \quad \bar{\mathbf{f}} = U_{14}/(\sqrt{3} r_1)\mathbf{f}, \quad \bar{\mathbf{h}}' = (U_{12} + U_{13}/\sqrt{3})/r_1\mathbf{h}', \\ r_1 &= U_{11}/V_{11} \quad r_2 = r_1 V_{15}/U_{14}, \\ r_3 &= r_1 (V_{12} - V_{13}/\sqrt{3})/(U_{12} + U_{13}/\sqrt{3}); \\ c_e &= (U_{12} - \sqrt{3}U_{13})/(U_{12} + U_{13}/\sqrt{3}), \\ c_\nu &= r_1(V_{12} + \sqrt{3}V_{13})/(U_{12} + U_{13}/\sqrt{3}). \end{aligned}$$

$$\mathbf{U}, \mathbf{V} : \mathbf{U}\mathbf{M}_D\mathbf{V}^T = \mathbf{M}_D \text{ diag}$$

Mass of lightest pair of doublet is $\sim$ weak scale (assume)

Neutrino Mass

The VEVs of the fields

$\Delta_R : (1, 1, 3)$ and $\Delta_L : (1, 3, 1)$ give neutrino Majorana masses.

The light neutrino mass is obtained as

$$\mathbf{m}_\nu^{\text{light}} = \mathbf{M}_L - \mathbf{M}_\nu^D \mathbf{M}_R^{-1} (\mathbf{M}_\nu^D)^T$$

Type I: Minkowski'77;
Yanagida'79,
Gellman, Ramond, Slansky
'79; Glashow'79;
Mohapatra, Senjanovic'80

where $\mathbf{M}_\nu^D = Y_h \langle H_u \rangle$, $\mathbf{M}_L = 2\sqrt{2}f \langle \Delta_L \rangle$, $\mathbf{M}_R = 2\sqrt{2}f \langle \Delta_R \rangle$.

Pure type II: $\mathbf{M}_L$ (In this talk)

Lazarides, Shafi, Wetterich,81,
Mohapatra, Senjanovic,81

Proton Decay

The proton decay is mediated by the colored Higgs triplets:

$$\varphi_T + \varphi_T^\dagger : ((3, 1, -1/3) + \text{c.c.}) ; \varphi_C + \varphi_C^\dagger : ((3, 1, -4/3) + \text{c.c.}).$$

These Higgs triplets appear in:

$$\mathbf{10} + \mathbf{120} + \mathbf{126} + \mathbf{126} + \overline{\mathbf{210}} \text{ multiplets.}$$

We generate both LLLL (CL) and RRRR (CR) operators:

$$-\mathbf{W5} = \mathbf{C}^{ijkl}_L \mathbf{q}_k \mathbf{q}_l \mathbf{q}_i \mathbf{\ell}_j + \mathbf{C}^{ijkl}_R \mathbf{e}^c_k \mathbf{u}^c_l \mathbf{u}^c_i \mathbf{d}^c_j$$

These operators are obtained by integrating out the triplet Higgs fields,

$\varphi_T = (H_T, D_T, D'_T, \Delta_T, \Delta_T^-, \Delta'^-, \phi_T)$ The fields with “'” are **decuplet**, and the others are **sextet or 15-plet** under SU(4) decomposition.

$$\varphi_C = (D_C, \Delta_C^-).$$

Proton Decay...

$$W^{\text{trip}}_Y = hH_T (q\bar{\ell} + u^c d^c) + hH_T (1/2 qq + e^c u^c) + f\Delta_T (q\bar{\ell} - u^c d^c) + \dots$$

...

$$C^{ijkl}_L = c h_{ij} h_{kl} + x_1 f_{ij} f_{kl} + x_2 h_{ij} f_{kl} + x_3 f_{ij} h_{kl} + x_4 h'_{ij} h_{kl} + x_5 h'_{ij} f_{kl},$$

$$C^{ijkl}_R = c h_{ij} h_{kl} + y_1 f_{ij} f_{kl} + y_2 h_{ij} f_{kl} + y_3 f_{ij} h_{kl} + y_4 h'_{ij} h_{kl} + \dots$$

$c = (M_T^{-1})_{11}$, and the other coefficients x_i, y_i are also given by the components of M_T^{-1} .

The proton decay amplitude:

$A = \alpha_2 \beta_p / (4\pi M_T m_{\text{SUSY}}) A_x$, where

$$A_x = c A_{hh} + x_1 A_{ff} + x_2 A_{hf} + x_3 A_{fh} + x_4 A_{h'h} + x_5 A_{h'f} + \dots$$

Proton Decay...

The current nucleon decay bounds,

$$|A_{p \rightarrow K \nu}| \leq 10^{-8}, \quad |A_{n \rightarrow \pi \nu}| \leq 10^{-8} \text{ and}$$

$|A_{n \rightarrow K \nu}| \leq 5 \cdot 10^{-8}$ for colored Higgsino mass is $2 \cdot 10^{16}$ GeV,
and squark and wino masses : 1 TeV and 250 GeV

One way to suppress the decay amplitude is by demanding cancellation among different terms.

In order to achieve that, **we need a cancellation among h , f and h'** to have small couplings

However, we also need cancellation among the same couplings to generate the large mass hierarchy among the quark masses

The best way to avoid the cancellation is to choose smaller values of $r_{2,3}$.

Proton Decay...

To suppress A_{hh} , the elements h_{11} and h_{22} (in h -diagonal basis) are needed to be suppressed rather than the up- and charm-quark Yukawa couplings, respectively. As a result, we need Yukawa texture to be $\bar{\mathbf{h}} \simeq \text{diag}(\sim \mathbf{0}, \sim \mathbf{0}, \mathbf{O}(1))$.

Once $\mathbf{h}$ is fixed, the other matrices $\mathbf{f}$ and $\mathbf{h}'$ are almost determined as

$$\bar{\mathbf{f}} = \begin{pmatrix} \sim \mathbf{0} & \sim \mathbf{0} & \lambda^3 \\ \sim \mathbf{0} & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda^2 \end{pmatrix} \quad \bar{\mathbf{h}}' = -i \begin{pmatrix} \mathbf{0} & \lambda^3 & \lambda^3 \\ -\lambda^3 & \mathbf{0} & \lambda^2 \\ -\lambda^3 & -\lambda^2 & \mathbf{0} \end{pmatrix}$$

where $\lambda \sim 0.2$.

Proton Decay...

One example for numerical fit for

$$\tan \beta(M_Z) = 50, \bar{h} = \text{diag}(0, 0, 0.638),$$

$$\bar{f} = \begin{pmatrix} \sim 0 & 0.0044 & 0.00208 \\ 0.0044 & 0.00945 & 0.0101 \\ 0.00208 & 0.0101 & 0.0071 \end{pmatrix} \quad \bar{h}' = i \begin{pmatrix} 0 & -0.0022 & 0.00046 \\ 0.0022 & 0 & 0.0181 \\ -0.00046 & -0.0181 & 0 \end{pmatrix}$$

$$r_1 = 0.966, r_2 = 0.135, r_3 = 0, |c_e| = 0.987.$$

$r_2 \neq 0$ to produce
correct charm

The coefficients, x_i, y_i , involve the colored Higgs mixings, which can be suppressed by our choice of the vacuum expectation values and the Higgs couplings.

The A_{hh} for $p \rightarrow K \bar{\nu}_\mu$ mode is $\sim 2 \cdot 10^{-11}$.

The Model Predictions

**The number of parameters in the models is 17
3 (h), 6(f), 3(h') and 5 Higgs parameters ($r_{1,2,3}$, c_e, c_ν).**

Explanation of the proton decay fix some parameters.
We choose $\bar{h}_{11,22} = 0$ and $r_3 = 0$.

Since we will be working pure type II seesaw, i.e., $M_\nu = f v_L$, c_ν is redundant in fitting fermion masses and mixings. This reduces the number of parameters to 13.

13 parameters: up-type quark masses, charged lepton masses, the CKM angles and the phase, the ratio of the squared of neutrino mass differences (m^2_{sol}/m^2_A), and the bi-maximal mixings as input parameters.

The down-type quark masses, U_{e3} and δ_{MNSP} etc are the predictions of this model.

Strange Quark Mass

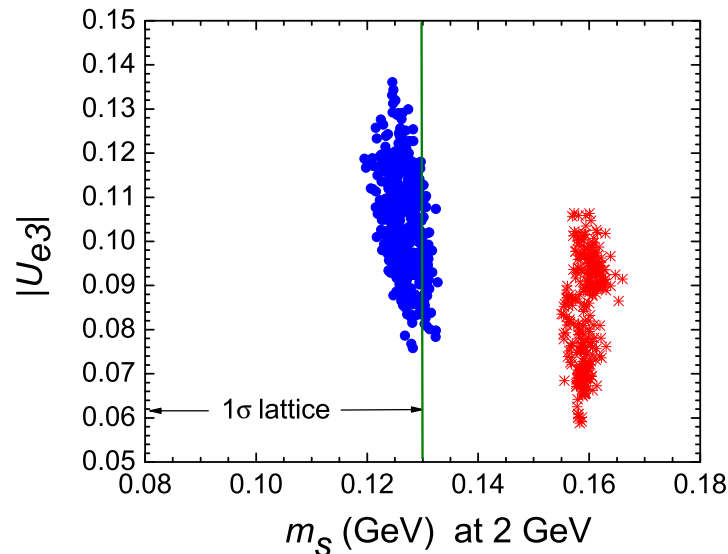
The predicted value of strange quark mass has two separate regions, roughly $m_s \sim 1/3 m_\mu (1 \pm O(\lambda^2))$.

The negative sign corresponds to a strange quark mass:

$$m_s(\mu = 2\text{GeV}) \sim 120\text{--}130 \text{ MeV}.$$

lattice derived value, $m_s(\mu = 2\text{GeV}) = (105 \pm 25) \text{ MeV}$

The positive signature gives the following value of the strange quark mass, $m_s(\mu = 2\text{GeV}) \sim 155\text{--}165 \text{ MeV}$



$$m_s/m_d = 17\text{--}18, 19\text{--}20.5$$

$[18.9 \pm 0.8, \text{Leutwyler'00}]$

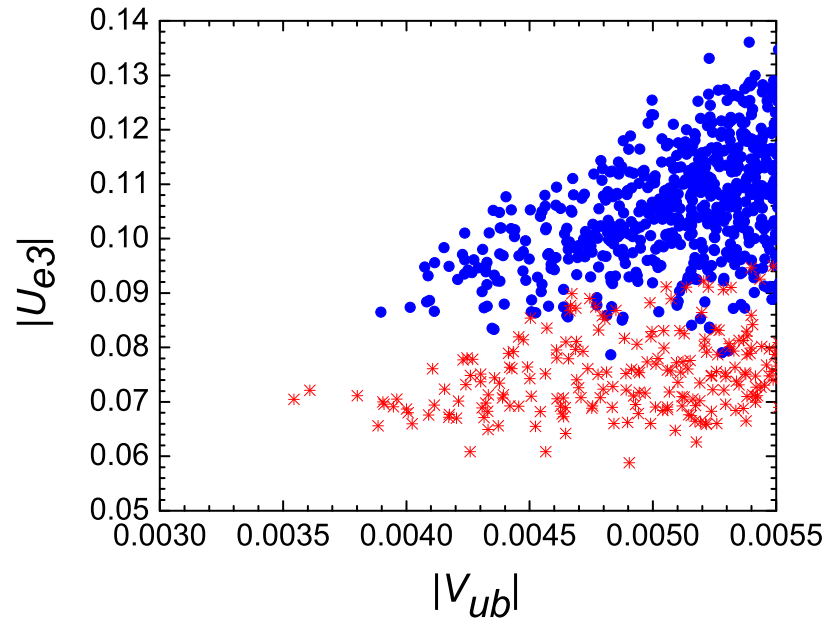
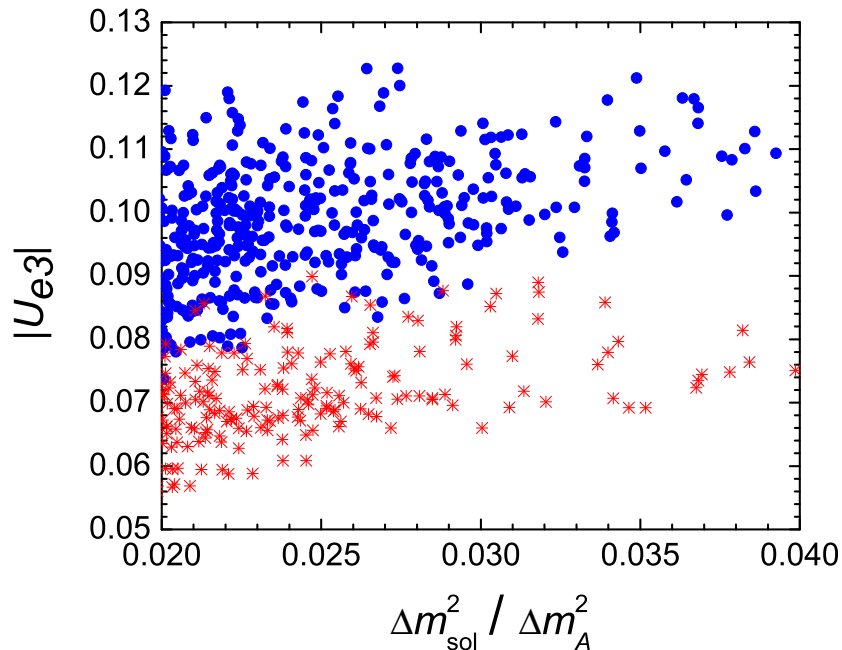
$$|U_{e3}|$$

We get the following approximate relation for U_{e3} :

$$|U_{e3}|^2 \approx \tan^2 \theta_{\text{sol}} / (1 - \tan^4 \theta_{\text{sol}}) \Delta m_{\text{sol}}^2 / \Delta m_A^2$$

We also have the following relation since U_{e3} is related to the ratio :

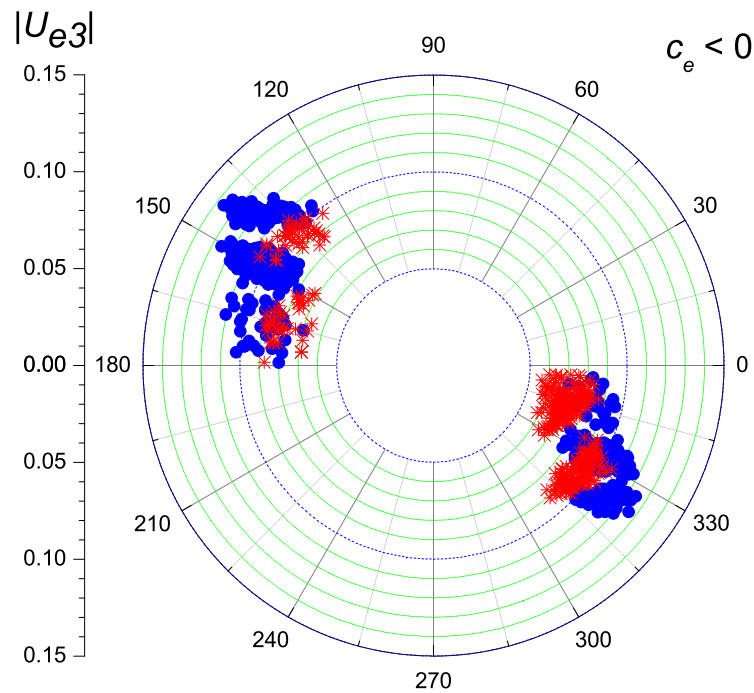
$$|U_{e3}| \approx 1/\sqrt{2} |V_{ub}/V_{cb}|$$



MNSP Phase

The MNSP phase is given by the approximate expression:

$$\sin \delta_{\text{MNSP}} \sim 1/\sqrt{2} \sin \theta_{12}^e \sin \theta_{13}^v \sin(\tan^{-1} c_e \bar{h}'_{12} / (3 \bar{f}_{12}))$$



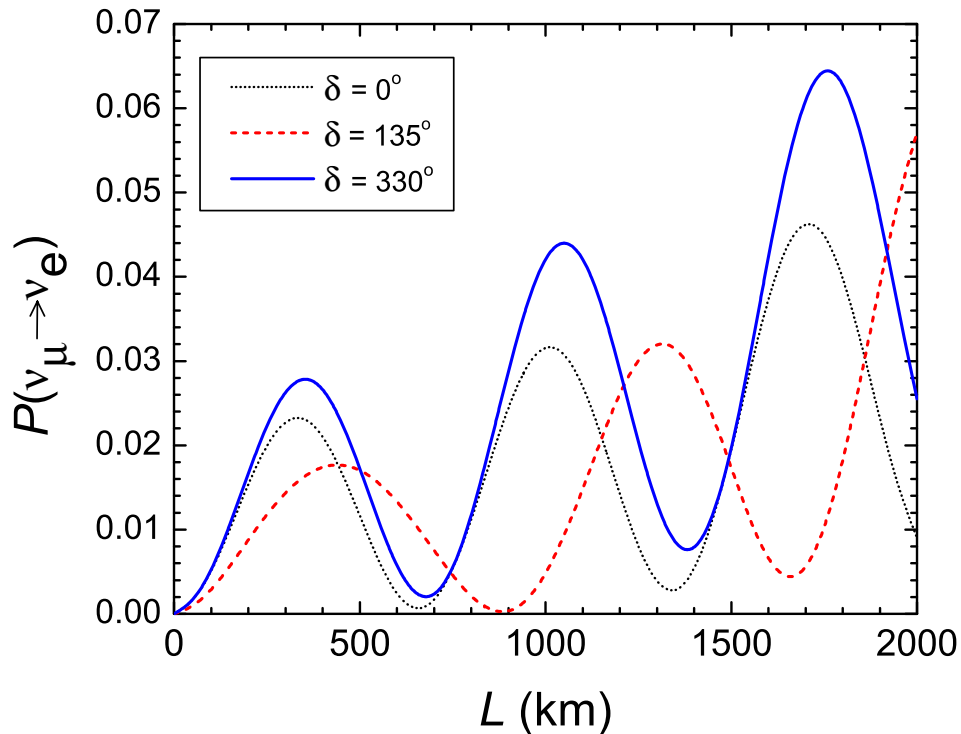
MNSP Phase

The location of δ_{MNSP} in the 2nd or 4th quadrant has impact on on the probability of ν_μ to ν_e oscillation ($P_{\mu \rightarrow e}$) which will be measured at the T2K experiment and at Tokai-to-Korea experiments.

This probability depends on sine and cosine of δ_{MNSP} , distance distance (L), energy of the neutrino beam, mass squared differences (m^2_{13} , m^2_{12}), 3 mixing angles, and matter density.

MNSP

We take the energy of the beam is 0.7 GeV and the values of mass squared differences: $m_{13}^2 = 2.5 \times 10^{-3} \text{ eV}^2$, $m_{12}^2 = 8 \times 10^{-5} \text{ eV}^2$ and $U_{e3} = 0.1$.



The probability for $\delta = 330^\circ$ is about **1.8** times bigger compared to the probability for $\delta = 135^\circ$ when the beam arrives at **Kamioka from Tokai ($L = 295 \text{ km}$)**. The difference is magnified much more if we have a detector installed at Korea ($L = 1000 \text{ km}$).

Lepton Flavor Violation

Lepton flavor violating processes, e.g., $\mu \rightarrow e\gamma$, $\tau \rightarrow \mu\gamma$ etc.

The operator for $l_i \rightarrow l_j + \gamma$:

$$\mathbf{L}_{li \rightarrow lj} = i(e/2m_l) l_j \sigma^{\mu\nu} \mathbf{q}_\nu (\mathbf{a}_l \mathbf{P}_L + \mathbf{a}_r \mathbf{P}_R) l_i \cdot \mathbf{A}_\mu + \text{h.c.}$$

$$\Gamma_{li \rightarrow lj + \gamma} = m_\mu (e^2 / 64\pi) (|\mathbf{a}_l|^2 + |\mathbf{a}_r|^2)$$

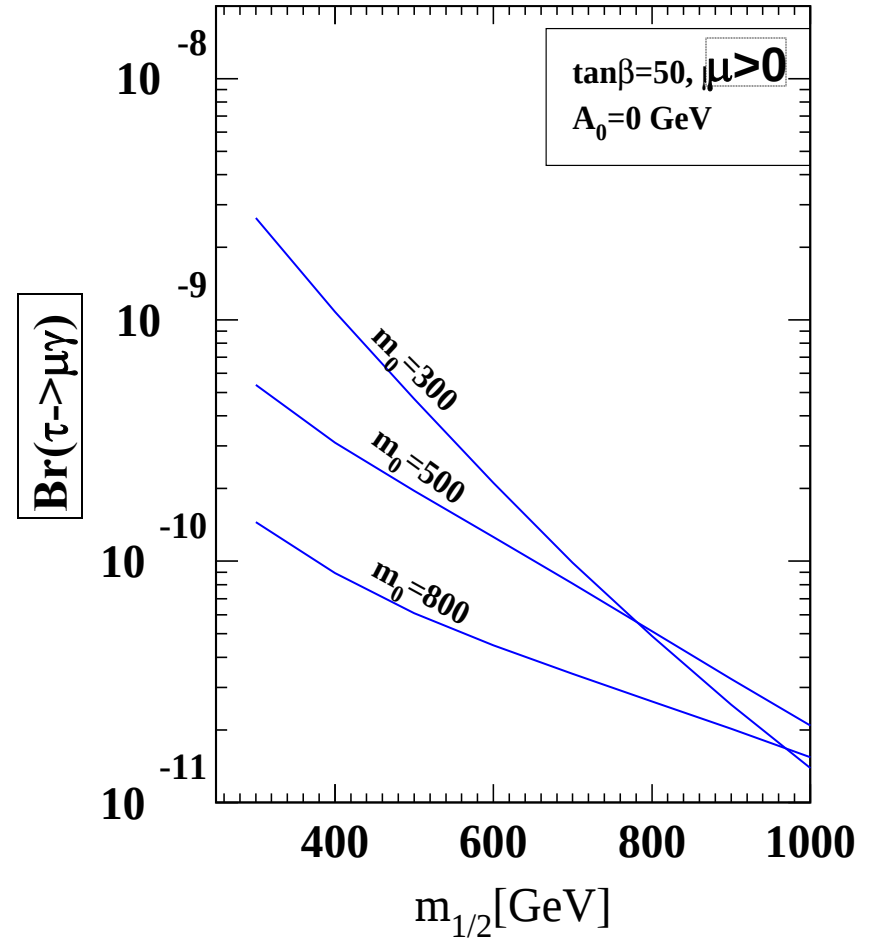
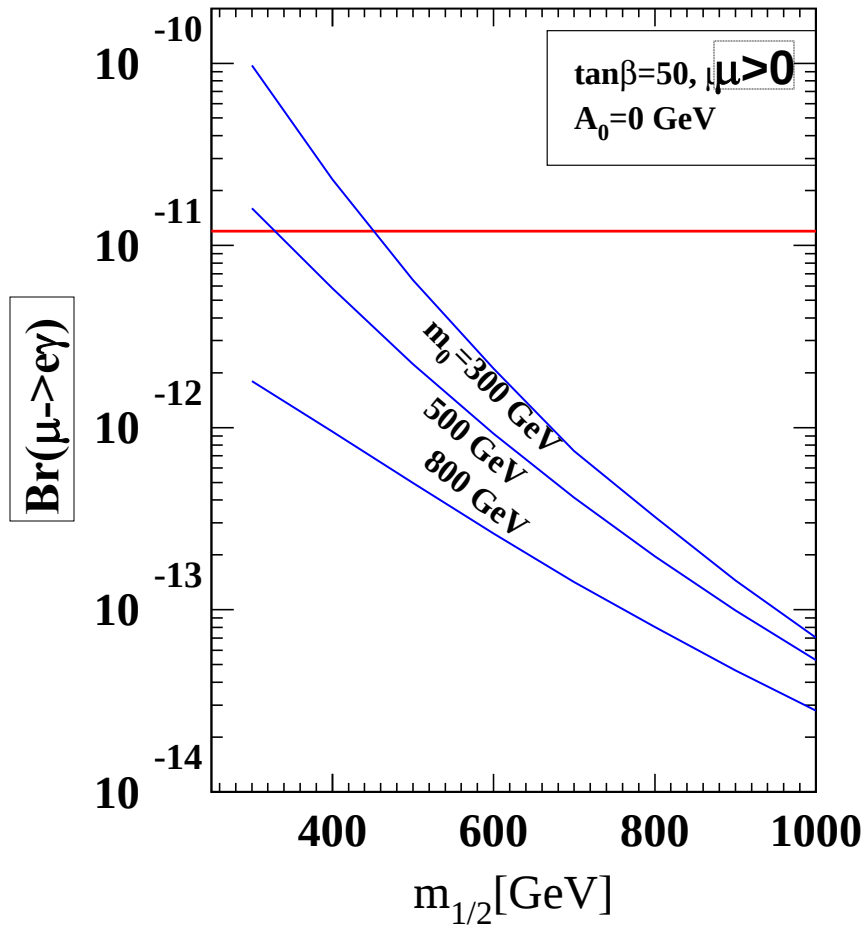
The right handed masses have hierarchies and therefore get decoupled at different scales.

The flavor-violating pieces present in Y induces flavor violations into the charged lepton couplings and into the soft SUSY breaking masses through the RGEs:

$$dY_e/dt = (1/16\pi^2) (Y_\nu Y_\nu^\dagger + \dots) Y_e$$

$$dm_{LL}^2/dt = (1/16\pi^2) (Y_\nu Y_\nu^\dagger m_{LL}^2 + m_{LL}^2 Y_\nu Y_\nu^\dagger + \dots)$$

Lepton Flavor Violation...



Conclusion

- ✓ We have constructed a realistic minimal SO(10) model
- ✓ The model suppresses proton decay naturally
- ✓ The fermion masses can be fit and the model has many predictions
- ✓ U_{e3} is about 0.1 (without fine tuning) in this model
- ✓ The model has interesting predictions for T2K experiments
- ✓ This model also will be detected in the upcoming results of $B_s \rightarrow \mu^+ \mu^-$. This BR is large since $\lambda_t \sim \lambda_b \sim \lambda_\tau$.