Neutrino Mixing, Majorana CP-Violation, Neutrinoless Double Beta Decay and Leptogenesis

S. T. Petcov

SISSA/INFN, Trieste, Italy, and
INRNE, Bulgarian Academy of Sciences, Sofia, Bulgaria

Search for Baryon and Lepton Number Violation
International Workshop
LBL, Berkeley, September 21, 2007

Low Energy Leptonic CPV and Leptogenesis: Summary

Leptogenesis: see-saw mechanism; N_j - heavy RH ν's;
N_j, ν_k - Majorana particles

N_j: $M_1 \ll M_2 \ll M_3$

The observed value of the baryon asymmetry of the Universe can be generated

A. CP-violation due to the Dirac phase δ in $U_{\rm PMNS}$, no other sources of CPV (Majorana phases in $U_{\rm PMNS}$ equal to 0, etc.); requires $M_1 \gtrsim 10^{11}$ GeV.

$m_1 \ll m_2 \ll m_3$ (NH):

$$|\sin\theta_{13}\sin\delta| \gtrsim 0.09\,, \qquad \sin\theta_{13} \gtrsim 0.09\,; \quad |J_{\rm CP}| \gtrsim 2.0 \times 10^{-2}$$

$m_3 \ll m_1 < m_2$ (IH):

$$|\sin\theta_{13}\sin\delta| \gtrsim 0.02\,, \qquad \sin\theta_{13} \gtrsim 0.02\,; \quad |J_{\rm CP}| \gtrsim 4.6 \times 10^{-3}$$

B. CP-violation due to the Majorana phases in $U_{\rm PMNS}$, no other sources of CPV (Dirac phase in $U_{\rm PMNS}$ equal to 0, etc.); requires $M_1 \gtrsim 3.5 \times 10^{10}$ GeV.

C. CP-violation due to both Dirac and Majorana phases in $U_{\rm PMNS}$.

D. Y_B can depend non-trivially on $\min(m_j) \sim (10^{-5} - 10^{-2})$ eV.

S. Pascoli, S.T.P., A. Riotto, 2006 (A-C);
E. Molinaro, S.T.P., T. Shindou, Y. Takanishi, 2007 (D).

Compelling Evidences for $\nu-$Oscillations: 3-ν mixing

$$\nu_{l\mathrm{L}} = \sum_{j=1} U_{lj}\, \nu_{j\mathrm{L}} \qquad l = e, \mu, \tau.$$

B. Pontecorvo, 1957; 1958; 1967;

Z. Maki, M. Nakagawa, S. Sakata, 1962;

Three Neutrino Mixing

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{3} U_{lj}\, \nu_{j\mathrm{L}} \ .$$

U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix,

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

- U - $n \times n$ unitary:

	n	2	3	4
mixing angles:	$\frac{1}{2}n(n-1)$	1	3	6
CP-violating phases:				
• $\nu_j -$ Dirac:	$\frac{1}{2}(n-1)(n-2)$	0	1	3
• $\nu_j -$ Majorana:	$\frac{1}{2}n(n-1)$	1	3	6

$n = 3$: 1 Dirac and
2 additional CP-violating phases, Majorana phases

S.M. Bilenky, J. Hosek, S.T.P.,1980;
J. Schechter, J.W.F. Valle,1980;
M. Doi, T. Kotani, E. Takasugi,1981

PMNS Matrix: Standard Parametrization

$$U = V \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix}$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- $s_{ij} \equiv \sin\theta_{ij},\ c_{ij} \equiv \cos\theta_{ij},\ \theta_{ij} = [0, \frac{\pi}{2}]$,

- δ - Dirac CP-violation phase, $\delta = [0, 2\pi]$,

- α_{21}, α_{31} - the two Majorana CP-violation phases.

- $\Delta m^2_{\odot} \equiv \Delta m^2_{21} \cong 8.0 \times 10^{-5}$ eV$^2 > 0$, $\sin^2\theta_{12} \cong 0.30$, $\cos 2\theta_{12} \gtrsim 0.28$ (2σ),

- $|\Delta m^2_{\text{atm}}| \equiv |\Delta m^2_{31}| \cong 2.5 \times 10^{-3}$ eV^2, $\sin^2 2\theta_{23} \cong 1$,

- θ_{13} - the CHOOZ angle: $\sin^2\theta_{13} < 0.027$ (0.041) 2σ (3σ).

A.Bandyopadhyay, S.Choubey, S.Goswami, S.T.P., D.P.Roy, hep-ph/0406328 (updated)

T. Schwetz, hep-ph/0606060; M. Maltoni et al, 2004 (v. 5)

- $\mathrm{sgn}(\Delta m^2_{\rm atm}) = \mathrm{sgn}(\Delta m^2_{31})$ not determined

$$\Delta m^2_{\rm atm} \equiv \Delta m^2_{31} > 0, \quad \text{normal mass ordering}$$

$$\Delta m^2_{\rm atm} \equiv \Delta m^2_{32} < 0, \quad \text{inverted mass ordering}$$

Convention: $m_1 < m_2 < m_3$ - NMO, $\quad m_3 < m_1 < m_2$ - IMO

$$m_1 \ll m_2 \ll m_3, \qquad \text{NH},$$

$$m_3 \ll m_1 < m_2, \qquad \text{IH},$$

$$m_1 \cong m_2 \cong m_3,\; m^2_{1,2,3} >> \Delta m^2_{\rm atm},\; \text{QD};\; m_j \gtrsim 0.10 \text{ eV}.$$

- Majorana phases α_{21}, α_{31}:
  - $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$ not sensitive;

S.M. Bilenky, J. Hosek, S.T.P.,1980;
P. Langacker, S.T.P., G. Steigman, S. Toshev, 1987

  - $|<m>|$ in $(\beta\beta)_{0\nu}$–decay depends on α_{21}, α_{31};
  - $\Gamma(\mu \to e + \gamma)$ etc. in SUSY theories depend on $\alpha_{21,31}$;
  - BAU, leptogenesis scenario: $\alpha_{21,31}$!

Future Progress

- Determination of the nature - Dirac or Majorana, of ν_j .

- Determination of sgn($\Delta m^2_{\rm atm}$), type of $\nu-$ mass spectrum

$$m_1 \ll m_2 \ll m_3, \qquad \text{NH},$$

$$m_3 \ll m_1 < m_2, \qquad \text{IH},$$

$$m_1 \cong m_2 \cong m_3,\ m^2_{1,2,3} >> \Delta m^2_{\rm atm},\ \text{QD};\ m_j \gtrsim 0.10\ \text{eV}.$$

- Determining, or obtaining significant constraints on, the absolute scale of ν_j-masses, or min(m_j).

- Status of the CP-symmetry in the lepton sector: violated due to δ (Dirac), and/or due to α_{21}, α_{31} (Majorana)?

- High precision determination of $\Delta m^2_{\odot}$, $\theta_{\odot}$, $\Delta m^2_{\rm atm}$, θ_{atm}.

- Measurement of, or improving by at least a factor of (5 - 10) the existing upper limit on, $\sin^2\theta_{13}$.

- Searching for possible manifestations, other than ν_l-oscillations, of the non-conservation of L_l, $l = e, \mu, \tau$, such as $\mu \rightarrow e + \gamma$, $\tau \rightarrow \mu + \gamma$, etc. decays.

• Understanding at fundamental level the mechanism giving rise to the $\nu-$ masses and mixing and to the L_l-non-conservation. Includes understanding

– the origin of the observed patterns of ν-mixing and ν-masses ;

– the physical origin of CPV phases in $U_{\rm PMNS}$;

– Are the observed patterns of ν-mixing and of $\Delta m^2_{21,31}$ related to the existence of a new symmetry?

– Is there any relations between $q-$mixing and $\nu-$ mixing? Is $\theta_{12} + \theta_c = \pi/4$?

– Is $\theta_{23} = \pi/4$, or $\theta_{23} > \pi/4$ or else $\theta_{23} < \pi/4$?

– Is there any correlation between the values of CPV phases and of mixing angles in $U_{\rm PMNS}$?

• Progress in the theory of ν-mixing might lead to a better understanding of the origin of the BAU.

– Can the Majorana and/or Dirac CPVP in $U_{\rm PMNS}$ be the leptogenesis CPV parameters at the origin of BAU?

Rephasing Invariants Associated with CPVP

Dirac phase δ:

$$J_{CP} = \mathrm{Im}\left\{U_{e1}\, U_{\mu 2}\, U^*_{e2}\, U^*_{\mu 1}\right\} \ .$$

C. Jarlskog, 1985 (for quarks)

CP-, T- violation effects in neutrino oscillations

P. Krastev, S.T.P., 1988

Majorana phases α_{21}, α_{31}:

$$S_1 = \mathrm{Im}\left\{U_{e1}U^*_{e3}\right\}, \quad S_2 = \mathrm{Im}\left\{U_{e2}U^*_{e3}\right\} \quad \text{(not unique); \quad or}$$
$$S'_1 = \mathrm{Im}\left\{U_{\tau 1}U^*_{\tau 2}\right\}, \quad S'_2 = \mathrm{Im}\left\{U_{\tau 2}U^*_{\tau 3}\right\}$$

J.F. Nieves and P. Pal, 1987, 2001
G.C. Branco et al., 1986
J.A. Aguilar-Saavedra and G.C. Branco, 2000

CP-violation: both $\mathrm{Im}\left\{U_{e1}U^*_{e3}\right\} \neq 0$ and $\mathrm{Re}\left\{U_{e1}U^*_{e3}\right\} \neq 0$.

S_1, S_2 appear in $|<m>|$ in $(\beta\beta)_{0\nu}$-decay.

In general, J_{CP}, S_1 and S_2 are independent.

Dirac CP-Nonconservation: δ in $U_{\rm PMNS}$

Observable manifestations in

$$\nu_l \leftrightarrow \nu_{l'}\ ,\quad \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}\ ,\qquad l,l' = e,\mu,\tau$$

- not sensitive to Majorana CPVP $\alpha_{21},\ \alpha_{31}$

CP-invariance:

N. Cabibbo, 1978
S.M. Bilenky, J. Hosek, S.T.P.,1980;
V. Barger et al.,1980.

$$P(\nu_l \to \nu_{l'}) = P(\bar{\nu}_l \to \bar{\nu}_{l'})\ ,\quad l \neq l' = e,\mu,\tau$$

CPT-invariance:

$$P(\nu_l \to \nu_{l'}) = P(\bar{\nu}_{l'} \to \bar{\nu}_l)$$

$$l = l':\quad P(\nu_l \to \nu_l) = P(\bar{\nu}_l \to \bar{\nu}_l)$$

T-invariance:

$$P(\nu_l \to \nu_{l'}) = P(\nu_{l'} \to \nu_l),\ l \neq l'$$

3ν—mixing:

$$A^{(l,l')}_{\rm CP} \equiv P(\nu_l \to \nu_{l'}) - P(\bar{\nu}_l \to \bar{\nu}_{l'})\ ,\quad l \neq l' = e,\mu,\tau$$

$$A^{(l,l')}_{\rm T} \equiv P(\nu_l \to \nu_{l'}) - P(\nu_{l'} \to \nu_l),\ l \neq l'$$

$$A^{(e,\mu)}_{\rm T} = A^{(\mu,\tau)}_{\rm T} = -A^{(e,\tau)}_{\rm T}$$

P.I. Krastev, S.T.P., 1988

In vacuum: $A_{\mathrm{T}}^{(e,\mu)} = J_{\mathrm{CP}} F_{osc}^{vac}$

$$J_{CP} = \mathrm{Im}\left\{U_{e1}\, U_{\mu 2}\, U_{e2}^{*}\, U_{\mu 1}^{*}\right\} = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos\theta_{13} \sin\delta$$

$$F_{osc}^{vac} = \sin(\frac{\Delta m_{21}^2}{2E}L) + \sin(\frac{\Delta m_{32}^2}{2E}L) + \sin(\frac{\Delta m_{13}^2}{2E}L)$$

P.I. Krastev, S.T.P., 1988

In matter: Matter effects violate

$$\mathrm{CP:} \qquad P(\nu_l \rightarrow \nu_{l'}) \neq P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'})$$

$$\mathrm{CPT:} \qquad P(\nu_l \rightarrow \nu_{l'}) \neq P(\bar{\nu}_{l'} \rightarrow \bar{\nu}_l)$$

P. Langacker et al., 1987

Can conserve the T-invariance **(Earth)**

$$P(\nu_l \rightarrow \nu_{l'}) = P(\nu_{l'} \rightarrow \nu_l),\ l \neq l'$$

In matter with constant density: $\mathrm{A_T^{(e,\mu)} = J_{CP}^{mat} F_{osc}^{mat}}$

$$\mathrm{J_{CP}^{mat} = J_{CP}^{vac}\ R_{CP}}$$

R_{CP} does not depend on θ_{23} and δ; $|R_{CP}| \lesssim 2.5$

P.I. Krastev, S.T.P., 1988

HOW?

- Reactor Experiments ~ 2 km

  $\sin 2\theta_{13}$

- Super Beams: θ_{13}, δ, ...

  JHF (T2K), SK (HK) 295 km

  NuMI (NOνA) $\sim$800 km

  SPL+$\beta-$beams, UNO (1 megaton):
  CERN-Frejus $\sim$140 km

  $\nu-$Factories $\sim$ 3000, 7000 km

If ν_j- Majorana particles, U_{PMNS} **contains (3-ν mixing)**

δ-Dirac, α_{21}, α_{31} - Majorana **physical CPV phases**

ν-oscillations $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$, $l, l' = e, \mu, \tau$,

- **are not sensitive to the nature of ν_j,**

S.M. Bilenky et al.,1980;
P. Langacker et al., 1987

- **provide information on $\Delta m^2_{jk} = m^2_j - m^2_k$, but not on the absolute values of ν_j masses.**

The Majorana nature of ν_j can manifest itself in the existence of $\Delta L = \pm 2$ processes:

$$K^+ \rightarrow \pi^- + \mu^+ + \mu^+$$
$$\mu^- + \ (\mathrm{A,Z}) \rightarrow \mu^+ + \ (\mathrm{A,Z}-2)$$

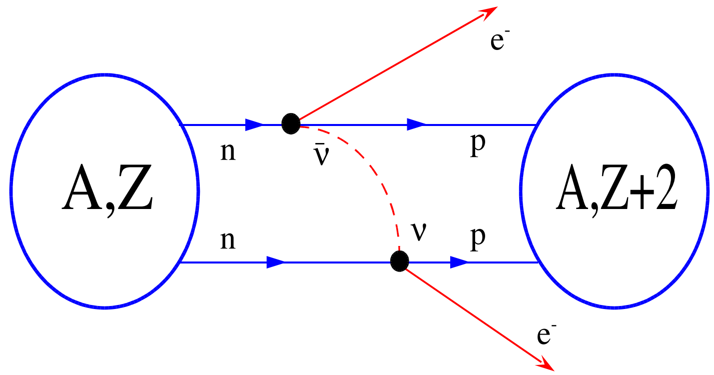
The process most sensitive to the possible Majorana nature of ν_j - $(\beta\beta)_{0\nu}$-decay

$$(\mathrm{A,Z}) \rightarrow (\mathrm{A,Z}+2) + e^- + e^-$$

of even-even nuclei, ^{48}Ca, ^{76}Ge, ^{82}Se, ^{100}Mo, ^{116}Cd, ^{130}Te, ^{136}Xe, ^{150}Nd.

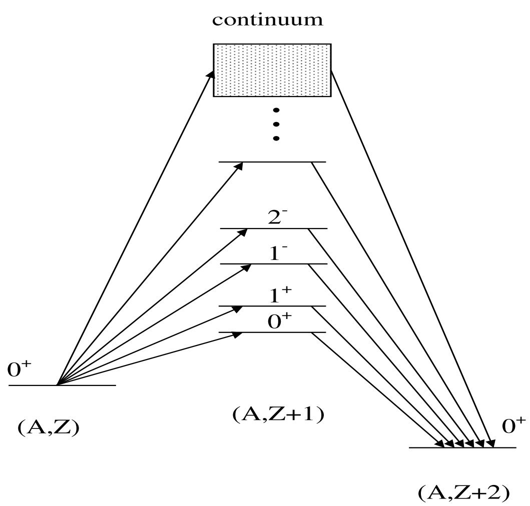
$2n$ from (A,Z) exchange a virtual Majorana ν_j (via the CC weak interaction) and transform into $2p$ of (A,Z+2) and two free e^-.

Nuclear $0\nu\beta\beta$-decay



strong in-medium modification of the basic process

$$dd \to uue^-e^-(\bar{\nu}_e\bar{\nu}_e)$$



virtual excitation
of states of all multipolarities
in (A,Z+1) nucleus

V. Rodin, talk at Gran Sasso, 2006

$(\beta\beta)_{0\nu}-$Decay Experiments:

- Majorana nature of ν_j
- Type of $\nu-$mass spectrum (NH, IH, QD)
- Absolute neutrino mass scale

^{3}H β-decay , cosmology: m_ν (QD, IH)

- CPV due to Majorana CPV phases

ν_j- Dirac or Majorana particles, **fundamental problem**

ν_j-Dirac: **conserved lepton charge exists,** $L = L_e + L_\mu + L_\tau$**,** $\nu_j \neq \bar{\nu}_j$

ν_j-Majorana: **no lepton charge is exactly conserved,** $\nu_j \equiv \bar{\nu}_j$

The observed patterns of $\nu-$mixing and of $\Delta m^2_{\rm atm}$ and $\Delta m^2_\odot$ can be related to Majorana ν_j and an approximate symmetry:

$$L' = L_e - L_\mu - L_\tau$$

S.T.P., 1982

See-saw mechanism: ν_j- Majorana

Establishing that ν_j are Majorana particles would be as important as the discovery of $\nu-$ oscillations.

$A(\beta\beta)_{0\nu} \sim < m >$ M(A,Z), M(A,Z) - NME,

$$|< m >| = \left|m_1|U_{e1}|^2 + m_2|U_{e2}|^2\ e^{i\alpha_{21}} + m_3|U_{e3}|^2\ e^{i\alpha_{31}}\right|$$

$$= \left|m_1\ c_{12}^2\ c_{13}^2 + m_2\ s_{12}^2\ c_{13}^2\ e^{i\alpha_{21}} + m_3\ s_{13}^2\ e^{i\alpha_{31}}\right|,\quad \theta_{12} \equiv \theta_{\odot},\ \theta_{13}\text{- CHOOZ}$$

α_{21}, α_{31} - the two Majorana CPVP of the PMNS matrix.

CP-invariance: $\alpha_{21} = 0, \pm\pi,\ \alpha_{31} = 0, \pm\pi;$

$$\eta_{21} \equiv e^{i\alpha_{21}} = \pm 1,\quad \eta_{31} \equiv e^{i\alpha_{31}} = \pm 1$$

relative CP-parities of ν_1 and ν_2, and of ν_1 and ν_3 .

L. Wolfenstein, 1981;

S.M. Bilenky, N. Nedelcheva, S.T.P., 1984;

B. Kayser, 1984.

$|{<}\,m\,{>}|$: m_j, $\theta_\odot \equiv \theta_{12}$, θ_{13}, $\alpha_{21,31}$

$m_{1,2,3}$ - in terms of min(m_j), $\Delta m^2_{\rm atm}$, $\Delta m^2_\odot$

S.T.P., A.Yu. Smirnov, 1994

Convention: $m_1 < m_2 < m_3$ - NMO, $m_3 < m_1 < m_2$ - IMO

$$\Delta m^2_\odot \equiv \Delta m^2_{21}, \quad m_2 = \sqrt{m_1^2 + \Delta m^2_\odot},$$

while either

$$\Delta m^2_{\rm atm} \equiv \Delta m^2_{31} > 0, \quad m_3 = \sqrt{m_1^2 + \Delta m^2_{\rm atm}}, \quad \text{normal mass ordering,} \quad \text{or}$$

$$\Delta m^2_{\rm atm} \equiv \Delta m^2_{32} < 0, \quad m_1 = \sqrt{m_3^2 + |\Delta m^2_{\rm atm}| - \Delta m^2_\odot}, \quad \text{inverted mass ordering}$$

The neutrino mass spectrum –

Normal hierarchical (NH) if $m_1 << m_2 << m_3$,

Inverted hierarchical (IH) if $m_3 << m_1 \cong m_2$,

Quasi-degenerate (QD) if $m_1 \cong m_2 \cong m_3 = m$, $m_j^2 >> |\Delta m^2_{\rm atm}|$; $m_j \gtrsim 0.1$ eV

Given $|\Delta m^2_{\rm atm}|$, $\Delta m^2_\odot$, $\theta_\odot$, θ_{13},

$$|{<}\,m\,{>}| = |{<}\,m\,{>}|\ (\mathrm{m_{min}}, \alpha_{21}, \alpha_{31}; \mathrm{S}), \ \mathrm{S} = \mathrm{NO(NH), IO(IH)}.$$

$A(\beta\beta)_{0\nu} \sim <m>$ M(A,Z), M(A,Z) - NME,

$$|<m>| \cong \left|\sqrt{\Delta m^2_{\odot}}\sin^2\theta_{12}e^{i\alpha} + \sqrt{\Delta m^2_{31}}\sin^2\theta_{13}e^{i\beta}\right| , \; m_1 \ll m_2 \ll m_3 \text{ (NH)},$$

$$|<m>| \cong \sqrt{m_3^2 + \Delta m^2_{13}}\left|\cos^2\theta_{12} + e^{i\alpha}\;\sin^2\theta_{12}\right| , \; m_3 < (\ll) m_1 < m_2 \text{ (IH)},$$

$$|<m>| \cong m\left|\cos^2\theta_{12} + e^{i\alpha}\;\sin^2\theta_{12}\right| , \; m_{1,2,3} \cong m \gtrsim 0.10 \text{ eV} \;\; \text{(QD)},$$

$\theta_{12} \equiv \theta_{\odot}$, θ_{13}-CHOOZ; $\alpha \equiv \alpha_{21}$, $\beta + 2\delta \equiv \alpha_{31}$.

CP-invariance: $\alpha = 0, \pm\pi$, $\beta_M = 0, \pm\pi$;

$$|<m>| \lesssim 5 \times 10^{-3} \text{ eV, NH;}$$

$$\sqrt{\Delta m^2_{13}}\cos 2\theta_{12} \cong 0.013 \text{ eV} \lesssim |<m>| \lesssim \sqrt{\Delta m^2_{13}} \cong 0.055 \text{ eV}, \quad \text{IH;}$$

$$m\cos 2\theta_{12} \lesssim |<m>| \lesssim m,\; m \gtrsim 0.10 \text{ eV}, \quad \text{QD} .$$

$\nu_\odot$, $\Delta m^2_{\mathrm{atm}}$, CHOOZ Data:

- $\theta_{12} = \theta_\odot \cong \frac{\pi}{6}$, $\qquad \theta_{23} = \theta_{\mathrm{atm}} \cong \frac{\pi}{4}$, $\qquad \theta_{13} < \frac{\pi}{12}$

$$U_{\mathrm{PMNS}} = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & \epsilon \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{2\sqrt{2}} & -\frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} .$$

Very different from the CKM-matrix!

- $\cos\theta_{12} \cong \cos(\frac{\pi}{4} - \frac{\pi}{12}) = \frac{1}{\sqrt{2}}(1+\lambda)$, $\quad \sin\theta_{12} \cong \frac{1}{\sqrt{2}}(1-\lambda)$,
- $\lambda \cong (0.20 - 0.25)$: $\quad \theta_\odot + \theta_c = \pi/4$?

Natural Possibility:

$$U = U^{\dagger}_{\text{lep}}(\lambda)\ U_{\text{bim(tri)}}$$

with

$$U_{\text{bim}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad U_{\text{tri}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}.$$

- $U^{\dagger}_{\text{lep}}(\lambda)$ - from diagonalization of the l^- mass matrix,
- $U_{\text{bim(tri)}}$ - from diagonalization of the $\nu-$mass matrix

Further, $\Delta m^2_{\odot} \ll |\Delta m^2_{\text{atm}}|$.

- U_{bim} can be associated with a symmetry:

$$L' = L_e - L_\mu - L_\tau$$

S.T.P., 1982

- $U_{\text{bim(tri)}}$ can be associated with a $\mu - \tau$ symmetry of M_ν

T. Fukuyama, H. Nishiura, 1997; R.N. Mohapatra, S. Nussinov, 1999;...

These symmetries cannot be exact.

The Case of CP Nonconservation

$M_{\text{lep}} = U_L^\dagger \, m_{\text{lep}}^{\text{diag}} \, U_R$, $M_\nu = U_\nu^T \, m_\nu^{\text{diag}} \, U_\nu$; $\quad U = e^{i\Phi} \, P \, \tilde{U} \, Q$

$$U_{\text{PMNS}} = U_{\text{L}}^\dagger \, U_\nu = \tilde{U}_{\text{lep}}^\dagger \; P_\nu \; \tilde{U}_\nu \; Q_\nu$$

- $\tilde{U}_{\text{lep}}$, $\tilde{U}_\nu$ - CKM-like: (3+3) angles, (1+1) CPVP
- $P_\nu = \text{diag}(1, e^{i\phi}, e^{i\omega})$, $Q_\nu = \text{diag}(1, e^{i\rho}, e^{i\sigma})$: 4 CPVP

U_{PMNS}: 3 angles, 3 CPVP

$\tilde{U}_{\text{lep}}^\dagger \; P_\nu \; \tilde{U}_\nu \; Q_\nu$: 6 angles, 6 CPVP; $\quad$ textures, symmetries

$$U_\nu = P\,\tilde{U}_\nu\,Q = \mathrm{diag}(1, e^{i\phi}, e^{i\omega})\,\tilde{U}_\nu\,\mathrm{diag}(1, e^{i\sigma}, e^{i\tau})$$

$$= P\,O_{23}(\theta^\nu_{23})\,U_{13}(\theta^\nu_{13}\,,\xi)\,O_{12}(\theta^\nu_{12})\,Q$$

$$= P \begin{pmatrix} c^\nu_{12}\,c^\nu_{13} & s^\nu_{12}\,c^\nu_{13} & s^\nu_{13}\,e^{-i\xi} \\ -s^\nu_{12}\,c^\nu_{23} - c^\nu_{12}\,s^\nu_{23}\,s^\nu_{13}\,e^{i\xi} & c^\nu_{12}\,c^\nu_{23} - s^\nu_{12}\,s^\nu_{23}\,s^\nu_{13}\,e^{i\xi} & s^\nu_{23}\,c^\nu_{13} \\ s^\nu_{12}\,s^\nu_{23} - c^\nu_{12}\,c^\nu_{23}\,s^\nu_{13}\,e^{i\xi} & -c^\nu_{12}\,s^\nu_{23} - s^\nu_{12}\,c^\nu_{23}\,s^\nu_{13}\,e^{i\xi} & c^\nu_{23}\,c^\nu_{13} \end{pmatrix} Q$$

$$\tilde{U}_\ell = O_{23}(\theta^\ell_{23})\,U_{13}(\theta^\ell_{13}, \psi)\,O_{12}(\theta^\ell_{12})$$

$$= \begin{pmatrix} c^\ell_{12}\,c^\ell_{13} & s^\ell_{12}\,c^\ell_{13} & s^\ell_{13}\,e^{-i\psi} \\ -s^\ell_{12}\,c^\ell_{23} - c^\ell_{12}\,s^\ell_{23}\,s^\ell_{13}\,e^{i\psi} & c^\ell_{12}\,c^\ell_{23} - s^\ell_{12}\,s^\ell_{23}\,s^\ell_{13}\,e^{i\psi} & s^\ell_{23}\,c^\ell_{13} \\ s^\ell_{12}\,s^\ell_{23} - c^\ell_{12}\,c^\ell_{23}\,s^\ell_{13}\,e^{i\psi} & -c^\ell_{12}\,s^\ell_{23} - s^\ell_{12}\,c^\ell_{23}\,s^\ell_{13}\,e^{i\psi} & c^\ell_{23}\,c^\ell_{13} \end{pmatrix},$$

$$U_{13}(\theta, \kappa) = \begin{pmatrix} \cos\theta & 0 & \sin\theta\,e^{-i\kappa} \\ 0 & 1 & 0 \\ -\sin\theta\,e^{i\kappa} & 0 & \cos\theta \end{pmatrix}$$

Suppose $\tilde{U}_\nu$ - bimaximal (real) and arises from

$$M_\nu = \frac{m}{\sqrt{2}} \left(\begin{array}{ccc} 0 & e^{-i\alpha'} & e^{-i\beta'} \\ e^{-i\alpha'} & 0 & \epsilon\ e^{-i\gamma'} \\ e^{-i\beta'} & \epsilon\ e^{-i\gamma'} & 0 \end{array} \right) ,$$

α', β', γ' - phases, $\epsilon \ll 1$

$\Delta m^2_{\text{atm}} \cong m^2$, $\Delta m^2_\odot \cong \sqrt{2}\epsilon\Delta m^2_{\text{atm}}$, $\epsilon \sim 0.025$, IH $\nu-$ masses

In the limit $\epsilon = 0$ and $U_{\text{lep}} = \mathbf{1}$,

$L' = L_e - L_\mu - L_\tau$ is conserved.

For $U_{\text{lep}} \neq \mathbf{1}$, $(\alpha' - \gamma')$, $(\beta' - \gamma')$ physical CPVP,

$Q_\nu = \mathbf{1}$, $P_\nu = \text{diag}(1, e^{i(\beta'-\gamma')}, e^{i(\alpha'-\gamma')})$

$U_{\text{PMNS}} = \tilde{U}^\dagger_{\text{lep}} P_\nu U_{\text{bimax}}$: 3 angles, 3 CPVP

In general, Dirac and Majorana CPV phases are independent.

However, for all $\sin^\ell \theta_{ij} \equiv \lambda_{ij} \lesssim \lambda$ small, in the model we are considering and to leading order in λ,

$$|{<}m{>}| \ \cong \sqrt{|\Delta m^2_{\text{atm}}|}\ |\cos 2\theta_\odot + i\, 8\, J_{CP}| \ .$$

P. Frampton, S.T.P., W. Rodejohann, 2004

Rephasing Invariants Associated with CPVP

Dirac phase δ:

$$J_{CP} = \text{Im}\left\{U_{e1}\,U_{\mu 2}\,U^*_{e2}\,U^*_{\mu 1}\right\}\ .$$

C. Jarlskog, 1985

CP-, T- violation effects in neutrino oscillations

P. Krastev, S.T.P., 1988

Majorana phases α_{21}, α_{31}:

$$S_1 = \text{Im}\{U_{e1}U^*_{e3}\}\,,\quad S_2 = \text{Im}\{U_{e2}U^*_{e3}\}\qquad \text{(not unique)}$$

J.F. Nieves and P. Pal, 1987, 2001
G.C. Branco et al., 1986
J.A. Aguilar-Saavedra and G.C. Branco, 2000

S_1, S_2 appear in $|<m>|$ in $(\beta\beta)_{0\nu}$-decay.

In general, J_{CP}, S_1 and S_2 are independent.

However, for, e.g., all $\sin^\ell\theta_{ij} \equiv \lambda_{ij} \lesssim \lambda$ small, in the model we are considering and to leading order in λ,

$$J_{CP} \simeq \frac{S_1}{2\sqrt{2}} \simeq \frac{S_2}{2\sqrt{2}}\ ,$$

and

$$|<m>|\ \cong \sqrt{|\Delta m^2_{\text{atm}}|}\ |\cos 2\theta_\odot + i\,8\,J_{CP}|\ .$$

P. Frampton, S.T.P., W. Rodejohann, 2004

Suppose further that M_ν has $\mu-\tau$ symmetry:

$$U_\nu = P\,\tilde{U}_\nu\,Q = \mathrm{diag}(1, e^{i\phi}, e^{i\omega})\,\tilde{U}_\nu\,\mathrm{diag}(1, e^{i\sigma}, e^{i\tau})$$

$$= P\,O_{23}(\theta^\nu_{23} = -\pi/4)\,U_{13}(\theta^\nu_{13} = 0\,, \xi)\,O_{12}(\theta^\nu_{12})\,Q$$

$$= P\,O_{23}(-\pi/4)\,O_{12}(\theta^\nu_{12})\,Q$$

$$\tilde{U}_\ell = O_{23}(\theta^\ell_{23} = 0)\,U_{13}(\theta^\ell_{13} = 0, \psi)\,O_{12}(\theta^\ell_{12}) = O_{12}(\theta^\ell_{12})$$

Now

$$U_{\mathrm{PMNS}} = \tilde{U}^\dagger_{\mathrm{lep}}\,U_\nu = O^T_{12}(\theta^\ell_{12})\,P\,O_{23}(-\pi/4)\,O_{12}(\theta^\nu_{12})\,Q$$

$$= \tilde{P}\,O_{12}(-\theta^\ell_{12})\,\mathrm{diag}(e^{-i\phi}, 1, 1)\,O_{23}(-\pi/4)\,O_{12}(\theta^\nu_{12})\,Q$$

$$\tilde{U}_\nu = O_{12}(\tilde{\theta}_{12})\,\mathrm{diag}(e^{-i\delta'}, 1, 1)\,O_{23}(\tilde{\theta}_{23})\,O_{12}(\theta'_{12})$$

$$= \begin{pmatrix} c'_{12}\,\tilde{c}_{12}\,e^{-i\delta'} - \tilde{c}_{23}\,s'_{12}\,\tilde{s}_{12} & \tilde{c}_{12}\,s'_{12}\,e^{-i\delta'} + c'_{12}\,\tilde{c}_{23}\,\tilde{s}_{12} & \tilde{s}_{12}\,\tilde{s}_{23} \\ -\tilde{c}_{12}\,\tilde{c}_{23}\,s'_{12} - c'_{12}\,\tilde{s}_{12}\,e^{-i\delta'} & c'_{12}\,\tilde{c}_{12}\,\tilde{c}_{23} - s'_{12}\,\tilde{s}_{12}\,e^{-i\delta'} & \tilde{c}_{12}\,\tilde{s}_{23} \\ s'_{12}\,\tilde{s}_{23} & -c'_{12}\,\tilde{s}_{23} & \tilde{c}_{23} \end{pmatrix}.$$

For $\sin^{\ell}\theta_{12} \equiv \lambda_{12}$ small, to leading order in λ,

$$\sin^2\theta_{12} = \sin^2\theta^{\nu}_{12} - \sin 2\theta^{\nu}_{12}\, |U_{e3}|\, \cos\phi \ ,$$

ϕ is the Dirac CPV phase,

$$\sin^2\theta_{12} = \sin^2\theta^{\nu}_{12} \pm \sqrt{|U_{e3}|^2 \sin^2 2\theta^{\nu}_{12} - 16\, J^2_{\mathrm{CP}}} \ ,$$

K. Hochmuth, S.T.P., W. Rodejohann, 2007

$$|{<}\,m\,{>}| \ \cong \sqrt{|\Delta m^2_{\mathrm{atm}}|}\ \sqrt{1 - \sin^2 2\theta_{\odot} \sin^2\sigma + 8\, J_{CP} \sin 2\sigma}\,, \quad \text{IH spectrum}\,,$$

$\sigma \equiv \alpha \equiv \alpha_{21}$ is Majorana CPV phase.

L. Everet, S.T.P., 2007

Best sensitivity: Heidelberg-Moscow ^{76}Ge experiment.

Claim for a positive signal at $> 3\sigma$:

H. Klapdor-Kleingrothaus et al., PL B586 (2004),

$|{<}\, m \,{>}| = (0.1 - 0.9)$ eV (99.73% C.L.).

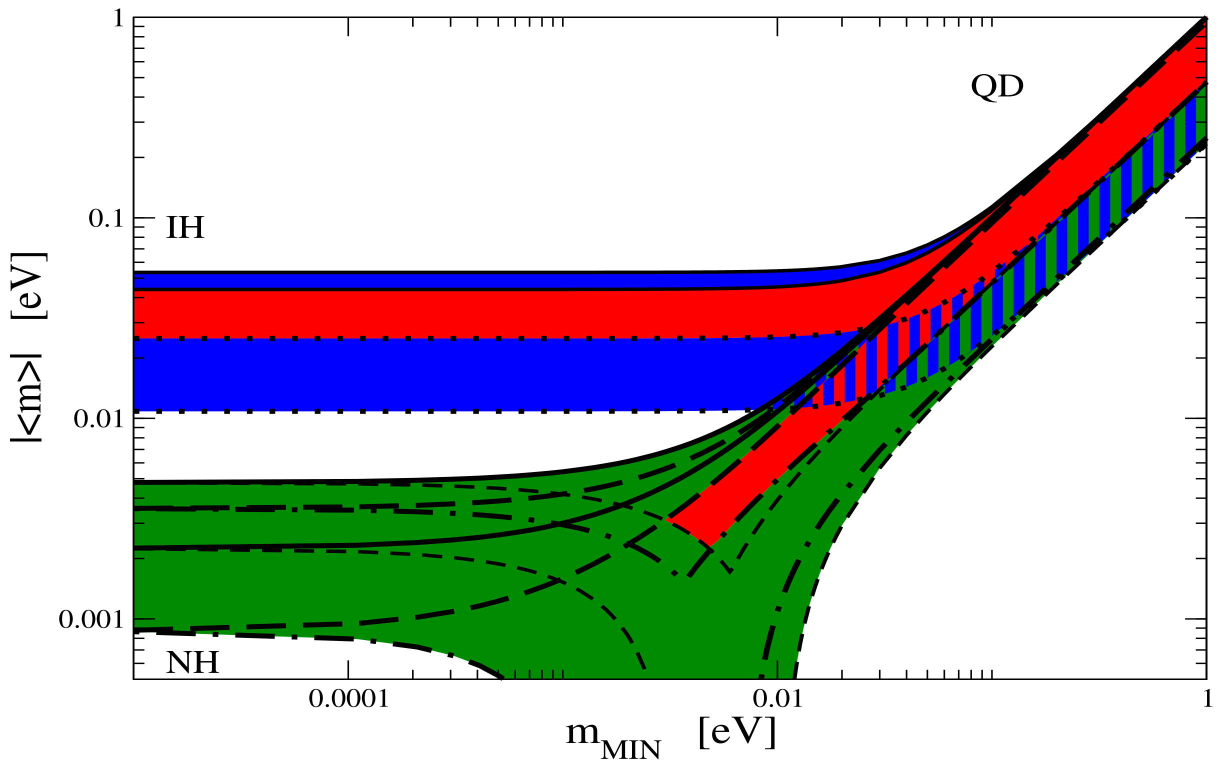
IGEX ^{76}Ge: $|{<}\, m \,{>}| < (0.33 - 1.35)$ eV (90% C.L.).

Taking data - NEMO3 (^{100}Mo), CUORICINO (^{130}Te):

$|{<}\, m \,{>}| <$(0.7–1.2) eV, $|{<}\, m \,{>}| <$(0.18–0.90) eV (90% C.L.).

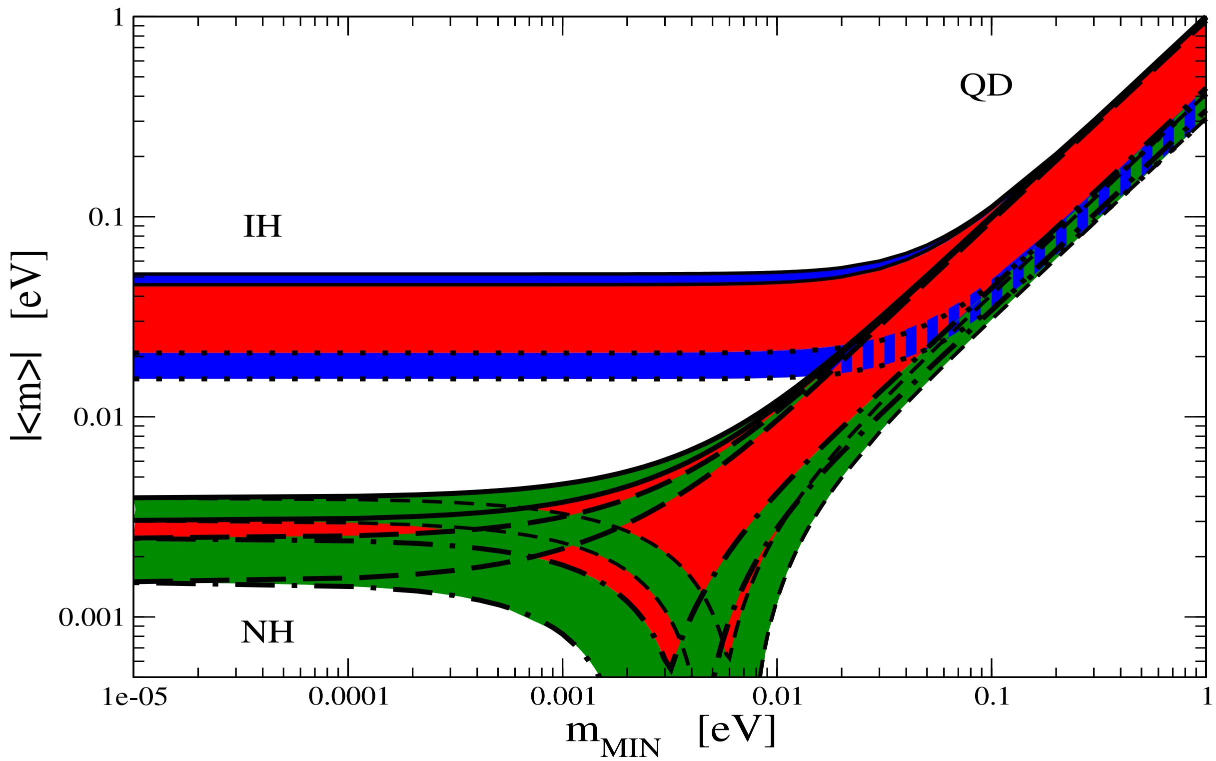
Large number of projects: $|{<}\, m \,{>}| \sim (0.01 - 0.05)$ eV

CUORE - ^{130}Te,
GERDA - ^{76}Ge,
SuperNEMO - ^{82}Se,
EXO - ^{136}Xe,
MAJORANA - ^{76}Ge,
MOON - ^{100}Mo,
CANDLES - ^{48}Ca,
XMASS - ^{136}Xe.



S. Pascoli, S.T.P., 2006

The current 2σ ranges of values of the parameters used.



$\sin^2\theta_{13} = 0.015 \pm 0.006$; $1\sigma(\Delta m^2_{\odot}) = 4\%$, $1\sigma(\sin^2\theta_{\odot}) = 4\%$, $1\sigma(|\Delta m^2_{\rm atm}|) = 6\%$;

$2\sigma(|{<}\,m\,{>}|\ )$ used.

Majorana CPV Phases and $|{<}\, m \,{>}|$

CPV can be established provided

– $|{<}\, m \,{>}|$ measured with $\Delta \lesssim 15\%$;

– $\Delta m^2_{\rm atm}$ (IH) or m_0 (QD) measured with $\delta \lesssim 10\%$;

– $\xi \lesssim 1.5$;

– α_{21} (QD): in the interval $\sim [\frac{\pi}{4} - \frac{3\pi}{4}]$, or $\sim [\frac{5\pi}{4} - \frac{3\pi}{2}]$;

– $\tan^2\theta_\odot \gtrsim 0.40$.

S. Pascoli, S.T.P., W. Rodejohann, 2002

S. Pascoli, S.T.P., L. Wolfenstein, 2002

S. Pascoli, S.T.P., T. Schwetz, hep-ph/0505226

No "No-go for detecting CP-Violation via $(\beta\beta)_{0\nu}$-decay"

V. Barger *et al.*, 2002

Absolute Neutrino Mass Measurements

The Troitzk and Mainz ^{3}H β-decay experiments

$$m_{\nu_e} < 2.3 \text{ eV} \qquad (95\% \text{ C.L.})$$

There are prospects to reach sensitivity

$$\text{KATRIN}: \qquad m_{\nu_e} \sim 0.2 \text{ eV}$$

Cosmological and astrophysical data: the WMAP result combined with data from large scale structure surveys (2dFGRS, SDSS)

$$\sum_j m_j \equiv \Sigma < (0.4 - 1.7) \text{ eV}$$

The WMAP and future PLANCK experiments can be sensitive to

$$\sum_j m_j \cong 0.4 \text{ eV}$$

Data on weak lensing of galaxies by large scale structure, combined with data from the WMAP and PLANCK experiments may allow to determine

$$\sum_j m_j : \qquad \delta \cong 0.04 \text{ eV}.$$

M_ν from the See-Saw Mechanism

P. Minkowski, 1977.
M. Gell-Mann, P. Ramond, R. Slansky, 1979;
T. Yanagida, 1979;
R. Mohapatra, G. Senjanovic, 1980.

- Explains the smallness of $\nu-$masses.

- Through **leptogenesis theory** links the $\nu-$mass generation to the generation of baryon asymmetry of the Universe Y_B.

S. Fukugita, T. Yanagida, 1986.

- In SUSY GUT's with see-saw mechanism of $\nu-$mass generation, the LFV decays

$$\mu \to e + \gamma, \quad \tau \to \mu + \gamma, \quad \tau \to e + \gamma \text{ , etc.}$$

are predicted to take place with rates within the reach of present and future experiments.

F. Borzumati, A. Masiero, 1986.

- The ν_j are **Majorana particles**; $(\beta\beta)_{0\nu}-$decay is allowed.

See-Saw: Dirac ν-mass m_D + Majorana mass M_R for N_R

The See-Saw Lagrangian

$$\mathcal{L}^{\text{lep}}(x) \;=\; \mathcal{L}_{\text{CC}}(x) + \mathcal{L}_{\text{Y}}(x) + \mathcal{L}_{\text{M}}^{\text{N}}(x)\,,$$

$$\begin{aligned}
\mathcal{L}_{\text{CC}} &= -\frac{g}{\sqrt{2}}\,\overline{l_L}(x)\,\gamma_\alpha\,\nu_{lL}(x)\,W^{\alpha\dagger}(x) + h.c.\,,\\
\mathcal{L}_{\text{Y}}(x) &= \lambda_{il}\,\overline{N_{iR}}(x)\,H^\dagger(x)\,\psi_{lL}(x) + Y_l\,H^c(x)\,\overline{l_R}(x)\,\psi_{lL}(x) + \text{h.c.}\,,\\
\mathcal{L}_{\text{M}}^{\text{N}}(x) &= -\frac{1}{2}\,M_i\,\overline{N_i}(x)\,N_i(x)\,.
\end{aligned}$$

ψ_{lL} - LH doublet, $\psi_{lL}^{\text{T}} = (\nu_{lL}\;\; l_L)$, l_R - RH singlet, H - Higgs doublet.
Basis: $M_R = (M_1, M_2, M_3)$; $D_N \equiv \text{diag}(M_1, M_2, M_3)$, $D_\nu \equiv \text{diag}(m_1, m_2, m_3)$.
m_D generated by the Yukawa interaction:

$$-\mathcal{L}_{\text{Y}}^{\nu} = \lambda_{il}\overline{N_{iR}}H^\dagger(x)\psi_{lL}(x),\;\; v = 174\text{ GeV},\;\; v\,\lambda = m_D - \text{complex}$$

For M_R - sufficiently large,

$$m_\nu \simeq v^2\;\lambda^T\,M_R^{-1}\,\lambda = U^*_{\text{PMNS}}\;m_\nu^{\text{diag}}\;U^\dagger_{\text{PMNS}}\;.$$

$Y_\nu \equiv \lambda = \sqrt{D_N}\;R\;\sqrt{D_\nu}\;(U_{\text{PMNS}})^\dagger/v_u$, all at M_R ; R-complex, $R^T R = \mathbf{1}$.

J.A. Casas and A. Ibarra, 2001

In GUTs, $M_R < M_X$, $M_X \sim 10^{16}$ GeV;

in GUTs, e.g., $M_R = (10^9, 10^{12}, 10^{15})$ GeV, $m_D \sim 1$ GeV.

The CP-Invarinace Constraints

Assume: $C(\overline{\nu}_j)^T = \nu_j, \;\; C(\overline{N}_k)^T = N_k, \;\; j,k = 1,2,3.$

The CP-symmetry transformation:

$$\begin{aligned} U_{\rm CP}\, N_j(x)\, U_{\rm CP}^\dagger &= \eta_j^{NCP}\, \gamma_0\, N_j(x')\,, \quad \eta_j^{NCP} = i\rho_j^N = \pm i\,, \\ U_{\rm CP}\, \nu_k(x)\, U_{\rm CP}^\dagger &= \eta_k^{\nu CP}\, \gamma_0\, \nu_k(x')\,, \quad \eta_k^{\nu CP} = i\rho_k^\nu = \pm i\,. \end{aligned}$$

CP-invariance:

$$\lambda^*_{jl} \;=\; \lambda_{jl}\, (\eta_j^{NCP})^*\, \eta^l\, \eta^{H*}\,, \quad j = 1,2,3,\; l = e,\mu,\tau,$$

Convenient choice: $\eta^l = i, \;\; \eta^H = 1 \quad (\eta^W = 1)$:

$$\begin{aligned} \lambda^*_{jl} &= \lambda_{jl}\, \rho_j^N\,, \;\; \rho_j^N = \pm 1\,, \\ U^*_{lj} &= U_{lj}\, \rho_j^\nu\,, \;\; \rho_j^\nu = \pm 1\,, \\ R^*_{jk} &= R_{jk}\, \rho_j^N\, \rho_k^\nu, \quad j,k = 1,2,3,\; l = e,\mu,\tau, \end{aligned}$$

λ_{jl}, U_{lj}, R_{jk} - either real or purely imaginary.

Relevant quantity:

$$\begin{aligned} P_{jkml} &\equiv R_{jk}\, R_{jm}\, U^*_{lk}\, U_{lm}\,, \;\; k \neq m\,, \\ CP: \quad P^*_{jkml} &= P_{jkml}\, (\rho_j^N)^2\, (\rho_k^\nu)^2\, (\rho_m^\nu)^2 = P_{jkml}\,, \quad \mathrm{Im}(P_{jkml}) = 0\,. \end{aligned}$$

$$
\begin{aligned}
P_{jkml} &\equiv R_{jk}\, R_{jm}\, U^*_{lk}\, U_{lm}\,, \;\; k \neq m\,, \\
CP: \quad P^*_{jkml} &= P_{jkml}\, (\rho^N_j)^2\, (\rho^\nu_k)^2\, (\rho^\nu_m)^2 = P_{jkml}\,, \quad \mathrm{Im}(P_{jkml}) = 0\,.
\end{aligned}
$$

Consider NH N_j, NH ν_k: $\quad P_{123\tau} = R_{12}\, R_{13}\, U^*_{\tau 2}\, U_{\tau 3}$

Suppose, CP-invrainace holds at low E: $\delta = 0$, $\alpha_{21} = \pi$, $\alpha_{31} = 0$.

Thus, $U^*_{\tau 2}\, U_{\tau 3}$ - purely imaginary.

Then real $R_{12}\, R_{13}$ corresponds to CP-violation at "high" E.

Leptogenesis

$$Y_B = \frac{n_B - n_{\bar{B}}}{S} \sim 8.6 \times 10^{-11} \qquad (n_\gamma\!: \sim 6.3 \times 10^{-10})$$

$$Y_B \cong -10^{-2} \;\; \varepsilon\, \kappa$$

W. Buchmüller, M. Plümacher, 1998;
W. Buchmüller, P. Di Bari, M. Plümacher, 2004

$\kappa-$ efficiency factor; $\kappa \sim 10^{-1} - 10^{-3}$: $\varepsilon \gtrsim 10^{-7}$.

ε: $CP-$, $L-$ violating asymmetry generated in out of equilibrium $N_{Rj}-$decays in the early Universe,

$$\varepsilon_1 = \frac{\Gamma(N_1 \to \Phi^- \ell^+) - \Gamma(N_1 \to \Phi^+ \ell^-)}{\Gamma(N_1 \to \Phi^- \ell^+) + \Gamma(N_1 \to \Phi^+ \ell^-)}$$

M.A. Luty, 1992;
L. Covi, E. Roulet and F. Vissani, 1996;
M. Flanz *et al.*, 1996;
M. Plümacher, 1997;
A. Pilaftsis, 1997.

$\kappa = \kappa(\widetilde{m})$, $\widetilde{m}$ - determines the rate of wash-out processes:

$\Phi^+ + \ell^- \to N_1$, $\ell^- + \Phi^+ \to \Phi^- + \ell^+$, etc.

W. Buchmuller, P. Di Bari and M. Plumacher, 2002;
G. F. Giudice *et al.*, 2004

Low Energy Leptonic CPV and Leptogenesis

Assume: $M_1 \ll M_2 \ll M_3$

Individual asymmetries:

$$\varepsilon_{1l} = -\frac{3M_1}{16\pi v^2} \frac{\text{Im}\left(\sum_{j,k} m_j^{1/2} m_k^{3/2} U_{lj}^* U_{lk} R_{1j} R_{1k}\right)}{\sum_j m_j \left|R_{1j}\right|^2}, \qquad v = 174 \text{ GeV}$$

$$\widetilde{m_l} \equiv \frac{|\lambda_{1l}|^2 v^2}{M_1} = \left|\sum_k R_{1k}\, m_k^{1/2}\, U_{lk}^*\right|^2, \quad l = e, \mu, \tau\,.$$

The "one-flavor" approximation - $\mathbf{Y}_{\mathbf{e},\mu,\tau}$ - "small":

Boltzmann eqn. for $n(N_1)$ and $\Delta L = \Delta(L_e + L_\mu + L_\tau)$.

$Y_l\; H^c(x)\overline{l_R}(x)\psi_{lL}$- out of equilibrium at $T \sim M_1$.

One-flavor approximation: $M_1 \sim T > 10^{12}$ GeV

$$\varepsilon_1 = \sum_l \varepsilon_{1l} = -\frac{3M_1}{16\pi v^2} \frac{\text{Im}\left(\sum_{j,k} m_j^2 R_{1j}^2\right)}{\sum_k m_k \left|R_{1k}\right|^2},$$

$$\widetilde{m_1} = \sum_l \widetilde{m_l} = \sum_k m_k \left|R_{1k}\right|^2.$$

Two-Flavour Regime

At $M_1 \sim T \sim 10^{12}$ GeV: Y_τ - in equilibrium, $Y_{e,\mu}$ - not; dynamics changes: τ_R^-, τ_L^+

$\tau_R^- + N_1 \rightarrow \nu_L + \tau_R^-, \quad N_1 + \nu_L \rightarrow \tau_R^- + \tau_L^+$, etc.

$\varepsilon_{1\tau}$ and $(\varepsilon_{1e} + \varepsilon_{1\mu}) \equiv \varepsilon_2$ evolve independently.

Three-Flavour Regime

At $M_1 \sim T \sim 10^9$ GeV: Y_τ, Y_μ - in equilibrium, Y_e - not.

$\varepsilon_{1\tau}$, ε_{1e} and $\varepsilon_{1\mu}$ evolve independently.

Thus, at $M_1 \sim 10^9 - 10^{12}$ GeV: L_τ, ΔL_τ - distinguishable;

L_e, L_μ, ΔL_e, ΔL_μ - individually not distinguishable;

$L_e + L_\mu$, $\Delta(L_e + L_\mu)$

A. Abada et al., 2006; E. Nardi et al., 2006

A. Abada et al., 2006

Individual asymmetries:

Assume: $M_1 \ll M_2 \ll M_3$, $10^9 \lesssim M_1\ (\sim T) \lesssim 10^{12}$ GeV,

$$\varepsilon_{1l} = -\frac{3M_1}{16\pi v^2}\,\frac{\mathrm{Im}\left(\sum_{j,k} m_j^{1/2} m_k^{3/2} U^*_{lj} U_{lk} R_{1j} R_{1k}\right)}{\sum_j m_j\, |R_{1j}|^2}$$

$$\widetilde{m_l} \equiv \frac{|\lambda_{1l}|^2\, v^2}{M_1} \;=\; \left|\sum_k R_{1k} m_k^{1/2} U^*_{lk}\right|^2 , \quad l = e, \mu, \tau\,.$$

The baryon asymmetry is

$$Y_B \;\simeq\; -\frac{12}{37 g_*}\left(\epsilon_2\, \eta\left(\frac{417}{589}\,\widetilde{m_2}\right) + \epsilon_\tau\, \eta\left(\frac{390}{589}\,\widetilde{m_\tau}\right)\right) ,$$

$$\eta\left(\widetilde{m_l}\right) \;\simeq\; \left(\left(\frac{\widetilde{m_l}}{8.25\times 10^{-3}\,\mathrm{eV}}\right)^{-1} + \left(\frac{0.2\times 10^{-3}\,\mathrm{eV}}{\widetilde{m_l}}\right)^{-1.16}\right)^{-1} .$$

$Y_{\mathcal{B}} = -(12/37)\,(Y_2 + Y_\tau)$,

$Y_2 = Y_{e+\mu}$, $\varepsilon_2 = \varepsilon_{1e} + \varepsilon_{1\mu}$, $\widetilde{m_2} = \widetilde{m_{1e}} + \widetilde{m_{1\mu}}$

A. Abada et al., 2006; E. Nardi et al., 2006

A. Abada et al., 2006

Real (Purely Imaginary) R: $\varepsilon_{1l} \neq 0$, CPV from U

$\varepsilon_{1e} + \varepsilon_{1\mu} + \varepsilon_{1\tau} = \varepsilon_2 + \varepsilon_{1\tau} = 0$,

$$
\begin{aligned}
\varepsilon_{1\tau} &= -\frac{3M_1}{16\pi v^2} \frac{\mathrm{Im}\left(\sum_{j,k} m_j^{1/2} m_k^{3/2} U^*_{\tau j} U_{\tau k} R_{1j} R_{1k}\right)}{\sum_j m_j \, |R_{1j}|^2} \\
&= -\frac{3M_1}{16\pi v^2} \frac{\sum_{j,k>j} m_j^{1/2} m_k^{1/2} (m_k - m_j)\, R_{1j} R_{1k} \mathrm{Im}\left(U^*_{\tau j} U_{\tau k}\right)}{\sum_j m_j \, |R_{1j}|^2}, \; R_{1j}R_{1k} = \pm \, |R_{1j}R_{1k}| \, , \\
&= \mp \frac{3M_1}{16\pi v^2} \frac{\sum_{j,k>j} m_j^{1/2} m_k^{1/2} (m_k + m_j)\, |R_{1j} R_{1k}| \, \mathrm{Re}\left(U^*_{\tau j} U_{\tau k}\right)}{\sum_j m_j \, |R_{1j}|^2}, \; R_{1j}R_{1k} = \pm i |R_{1j}R_{1k}|
\end{aligned}
$$

S. Pascoli, S.T.P., A. Riotto, 2006.

CP-Violation: $\mathrm{Im}\left(U^*_{\tau j} U_{\tau k}\right) \neq 0$, $\mathrm{Re}\left(U^*_{\tau j} U_{\tau k}\right) \neq 0$;

$$
Y_B = -\frac{12}{37} \frac{\varepsilon_{1\tau}}{g_*} \left(\eta\left(\frac{390}{589}\widetilde{m_\tau}\right) - \eta\left(\frac{417}{589}\widetilde{m_2}\right)\right)
$$

$m_1 \ll m_2 \ll m_3,\ M_1 \ll M_{2,3};\quad R_{12}R_{13}$ – real; $\quad m_1 \cong 0,\ R_{11} \cong 0$ (N_3 decoupling)

$$\varepsilon_{1\tau} = -\frac{3M_1\sqrt{\Delta m^2_{31}}}{16\pi v^2}\left(\frac{\Delta m^2_{\odot}}{\Delta m^2_{31}}\right)^{\frac{1}{4}} \frac{|R_{12}R_{13}|}{\left(\frac{\Delta m^2_{\odot}}{\Delta m^2_{31}}\right)^{\frac{1}{2}} |R_{12}|^2 + |R_{13}|^2} \times \left(1 - \frac{\sqrt{\Delta m^2_{\odot}}}{\sqrt{\Delta m^2_{31}}}\right) \mathrm{Im}\left(U^*_{\tau 2}U_{\tau 3}\right)$$

$$\mathrm{Im}\left(U^*_{\tau 2}U_{\tau 3}\right) = -c_{13}\left[c_{23}s_{23}c_{12}\sin\left(\frac{\alpha_{32}}{2}\right) - c^2_{23}s_{12}s_{13}\sin\left(\delta - \frac{\alpha_{32}}{2}\right)\right]$$

$\alpha_{32} = \pi,\ \delta = 0$: $\quad \mathrm{Re}(U^*_{\tau 2}U_{\tau 3}) = 0,\qquad$ CPV due to R

S. Pascoli, S.T.P., A. Riotto, 2006.

$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$ (NH)

Dirac CP-violation

$\alpha_{32} = 0\ (2\pi)$, $\beta_{23} = \pi\ (0)$; $\beta_{23} \equiv \beta_{12} + \beta_{13} \equiv \arg(R_{12}R_{13})$.

$|R_{12}|^2 \cong 0.85$, $|R_{13}|^2 = 1 - |R_{12}|^2 \cong 0.15$ - maximise $|\epsilon_\tau|$ and $|Y_B|$:

$$|Y_B| \cong 2.8 \times 10^{-13}\, |\sin\delta| \left(\frac{s_{13}}{0.2}\right) \left(\frac{M_1}{10^9\ \text{GeV}}\right) .$$

$|Y_B| \gtrsim 8 \times 10^{-11}$, $\quad M_1 \lesssim 5 \times 10^{11}$ GeV imply

$$|\sin\theta_{13}\sin\delta| \gtrsim 0.11\,, \qquad \sin\theta_{13} \gtrsim 0.11\,.$$

The lower limit corresponds to

$$|J_{\text{CP}}| \gtrsim 2.4 \times 10^{-2}$$

FOR $\alpha_{32} = 0\ (2\pi)$, $\beta_{23} = 0\ (\pi)$:

$$|\sin\theta_{13}\sin\delta| \gtrsim 0.09\,, \qquad \sin\theta_{13} \gtrsim 0.09\,; \qquad |J_{\text{CP}}| \gtrsim 2.0 \times 10^{-2}$$

$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$ (NH)

Majorana CP-violation

$\delta = 0$, real R_{12}, R_{13} ($\beta_{23} = \pi$ (0));

$\alpha_{32} \cong \pi/2$, $|R_{12}|^2 \cong 0.85$, $|R_{13}|^2 = 1 - |R_{12}|^2 \cong 0.15$ - maximise $|\epsilon_\tau|$ and $|Y_B|$:

$$|Y_B| \cong 2 \times 10^{-12} \left(\frac{\sqrt{\Delta m^2_{31}}}{0.05 \text{ eV}} \right) \left(\frac{M_1}{10^9 \text{ GeV}} \right) .$$

We get $|Y_B| \gtrsim 8 \times 10^{-11}$, for $M_1 \gtrsim 3.6 \times 10^{10}$ GeV

$M_1 \ll M_2 \ll M_3$, $m_3 \ll m_1 < m_2$ (IH)

$m_3 \cong 0$, $R_{13} \cong 0$ (N_3 decoupling): impossible to reproduce Y_B^{obs} for real $R_{11}R_{12}$

Dirac CP-violation, purely imaginary $R_{11}R_{12}$

$\alpha_{21} = \pi$; $R_{11}R_{12} = i\kappa|R_{11}R_{12}|$, $\kappa = 1$;

$|R_{11}| \cong 1.07$, $|R_{12}|^2 = |R_{11}|^2 - 1$, $|R_{12}| \cong 0.38$ - maximise $|\epsilon_\tau|$ and $|Y_B|$:

$$|Y_B| \cong 8.1 \times 10^{-12} \, |s_{13} \sin\delta| \left(\frac{M_1}{10^9 \text{ GeV}}\right).$$

$|Y_B| \gtrsim 8 \times 10^{-11}$, $M_1 \lesssim 5 \times 10^{11}$ GeV imply

$$|\sin\theta_{13} \sin\delta| \gtrsim 0.02\,, \qquad \sin\theta_{13} \gtrsim 0.02\,.$$

The lower limit corresponds to

$$|J_{\text{CP}}| \gtrsim 4.6 \times 10^{-3}$$

$M_1 \ll M_2 \ll M_3$, $m_3 \ll m_1 < m_2$ (IH)

Majorana or Dirac CP-violation

$m_3 \neq 0$, $R_{13} \neq 0$, $R_{11}(R_{12}) = 0$: possible to reproduce Y_B^{obs} for real $R_{12(11)}R_{13} \neq 0$

Requires $m_3 \cong (10^{-5} - 10^{-2})$ eV; non-trivial dependence of $|Y_B|$ on m_3

Majorana CPV, $\delta = 0$ (π): requires $M_1 \gtrsim 3.5 \times 10^{10}$ GeV

Dirac CPV, $\alpha_{32(31)} = 0$: typically requires $M_1 \gtrsim 10^{11}$ GeV

$|Y_B| \gtrsim 8 \times 10^{-11}$, $\quad M_1 \lesssim 5 \times 10^{11}$ GeV imply

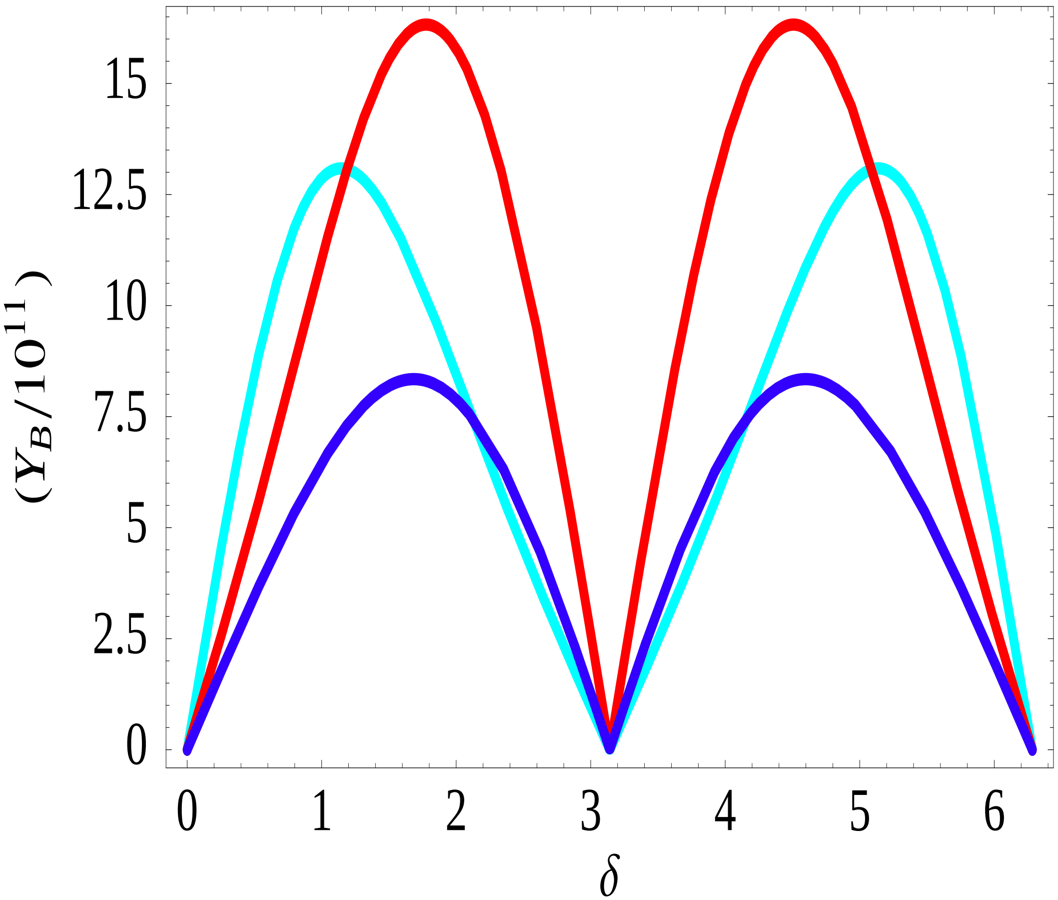
$$|\sin\theta_{13}\, \sin\delta|,\;\; \sin\theta_{13} \gtrsim (0.04 - 0.09)\,.$$

The lower limit corresponds to

$$|J_{\mathrm{CP}}| \gtrsim (0.009 - 0.02)$$

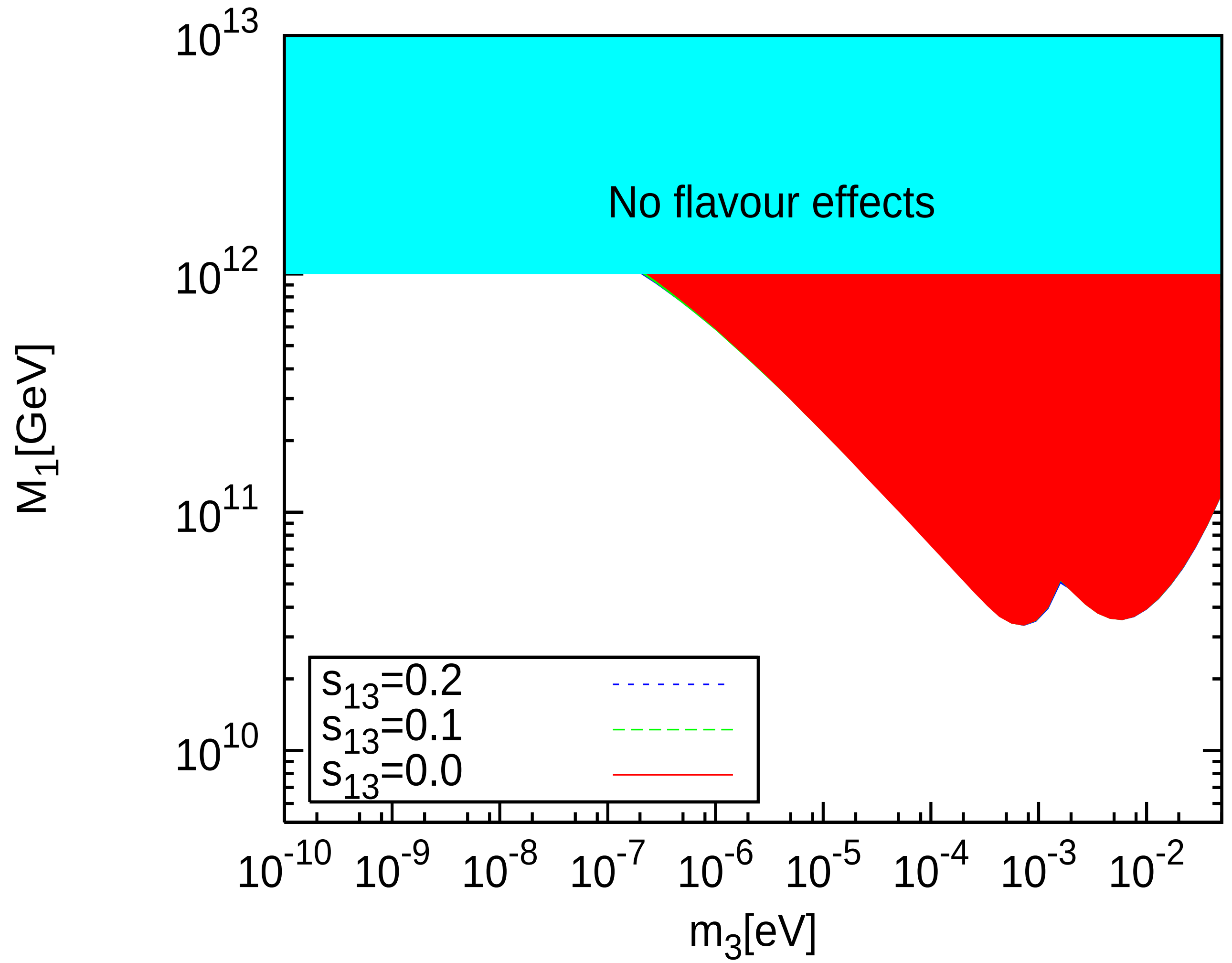
NO (NH) spectrum, $m_1 <\ (\ll)\ m_2 < m_3$: similar dependence of $|Y_B|$ on m_1 if $R_{12} = 0$, $R_{11}R_{13} \neq 0$; non-trivial effects for $m_1 \cong (10^{-5} - 5 \times 10^{-2})$ eV.

E. Molinaro, S.T.P., T. Shindou, Y. Takanishi, 2007



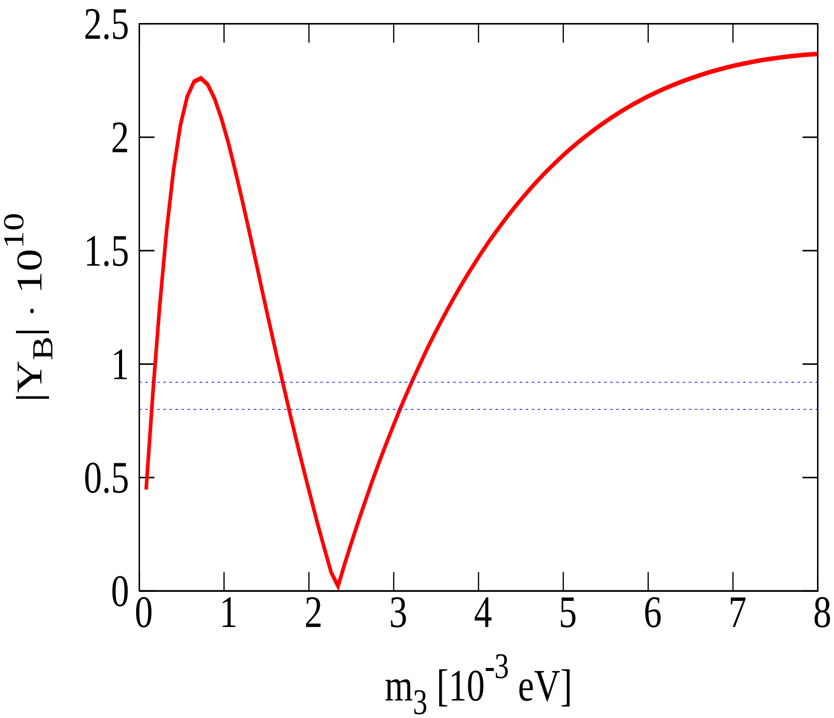
$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$; Dirac CP-violation, $\alpha_{32} = 0$; 2π;
real R_{12}, R_{13}, $|R_{12}|^2 + |R_{13}|^2 = 1$, $|R_{12}| = 0.86$, $|R_{13}| = 0.51$, sign$(R_{12}R_{13}) = +1$;
i) $\alpha_{32} = 0$ ($\kappa' = +1$), $s_{13} = 0.2$ (red line) and $s_{13} = 0.1$ (dark blue line);
ii) $\alpha_{32} = 2\pi$ ($\kappa' = -1$), $s_{13} = 0.2$ (light blue line);
$M_1 = 5 \times 10^{11}$ GeV.

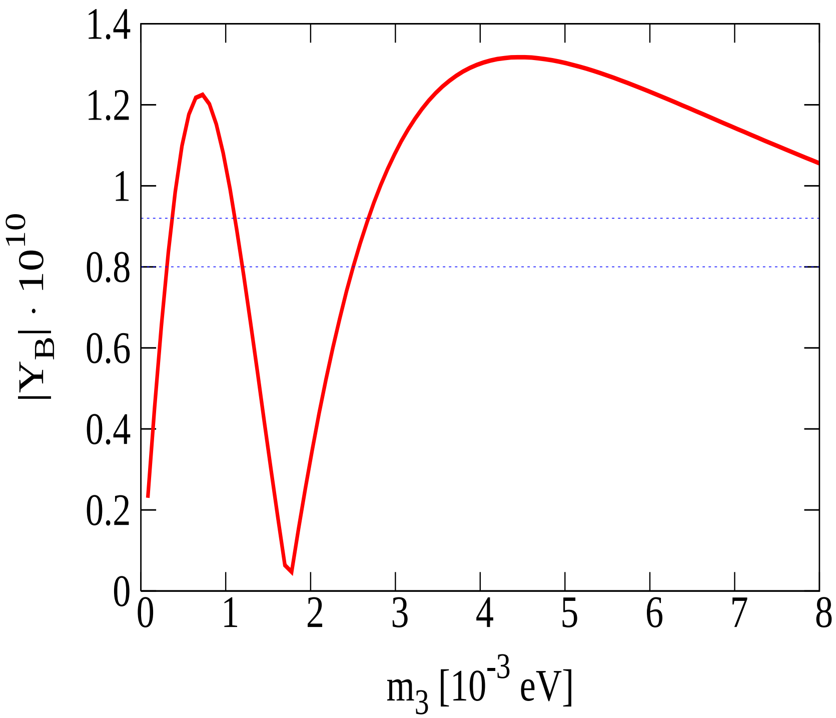
S. Pascoli, S.T.P., A. Riotto, 2006.



$m_3 < m_1 < m_2$, $M_1 \ll M_2 \ll M_3$, real R_{1j}; $M_1 = (10^9 - 10^{12})$ GeV, $s_{13} = 0.2; 0.1; 0$;
R_{1j} varied within $|R_{13}|^2 + |R_{12}|^2 + |R_{13}|^2 = 1$; $\alpha_{21}, \alpha_{31}, \delta$ varied in $[0, 2\pi]$;
min(M_1) for given m_3: $|Y_B| = 8.6 \times 10^{-11}$; absolute minima of M_1:
$m_3 \cong 5.5 \times 10^{-4}$; 5.9×10^{-3} eV, $\alpha_{32} \cong \pi/2$, $M_1 = 3.4\ (3.5) \times 10^{10}$ GeV.

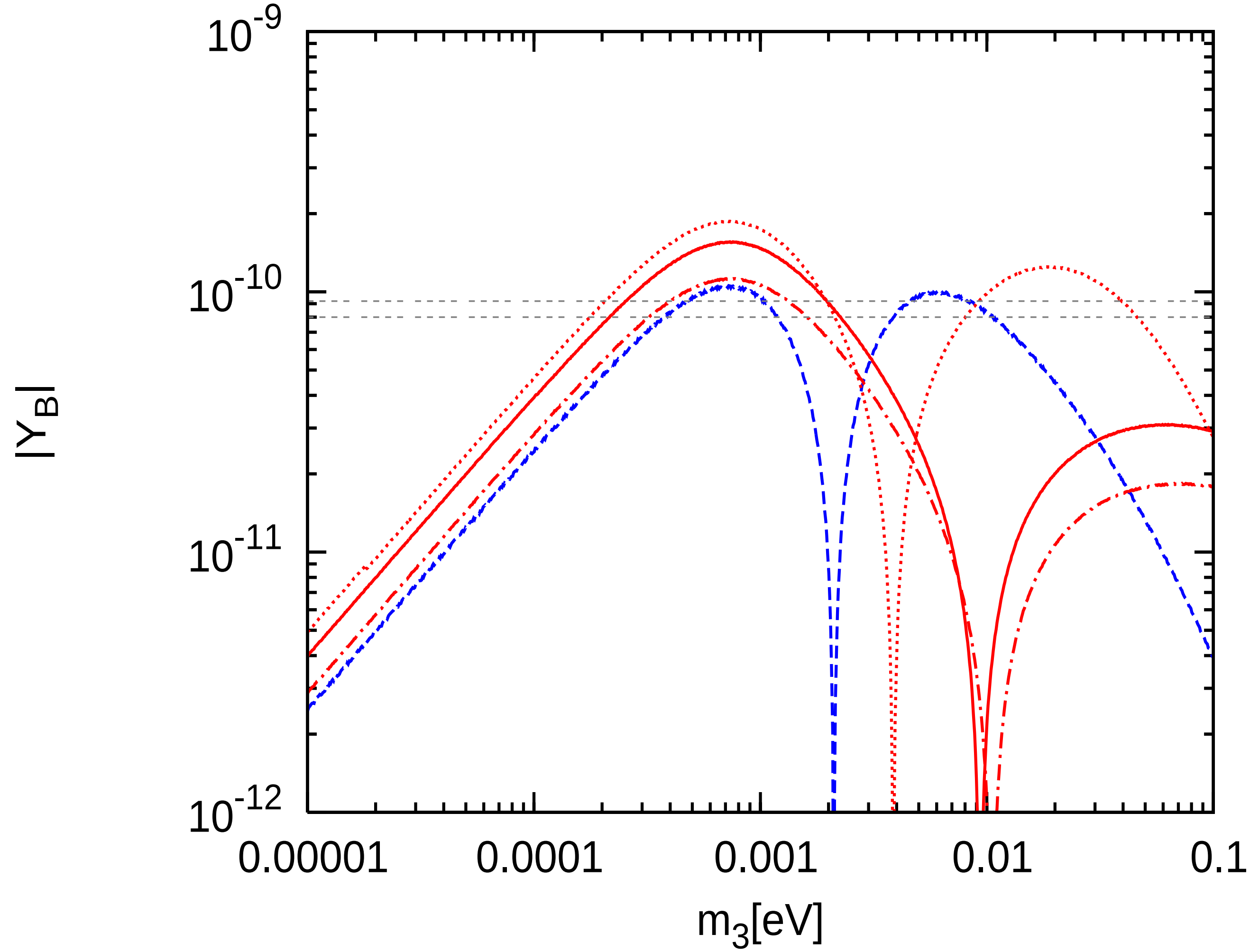
E. Molinaro, S.T.P., T. Shindou, Y. Takanishi, 2007





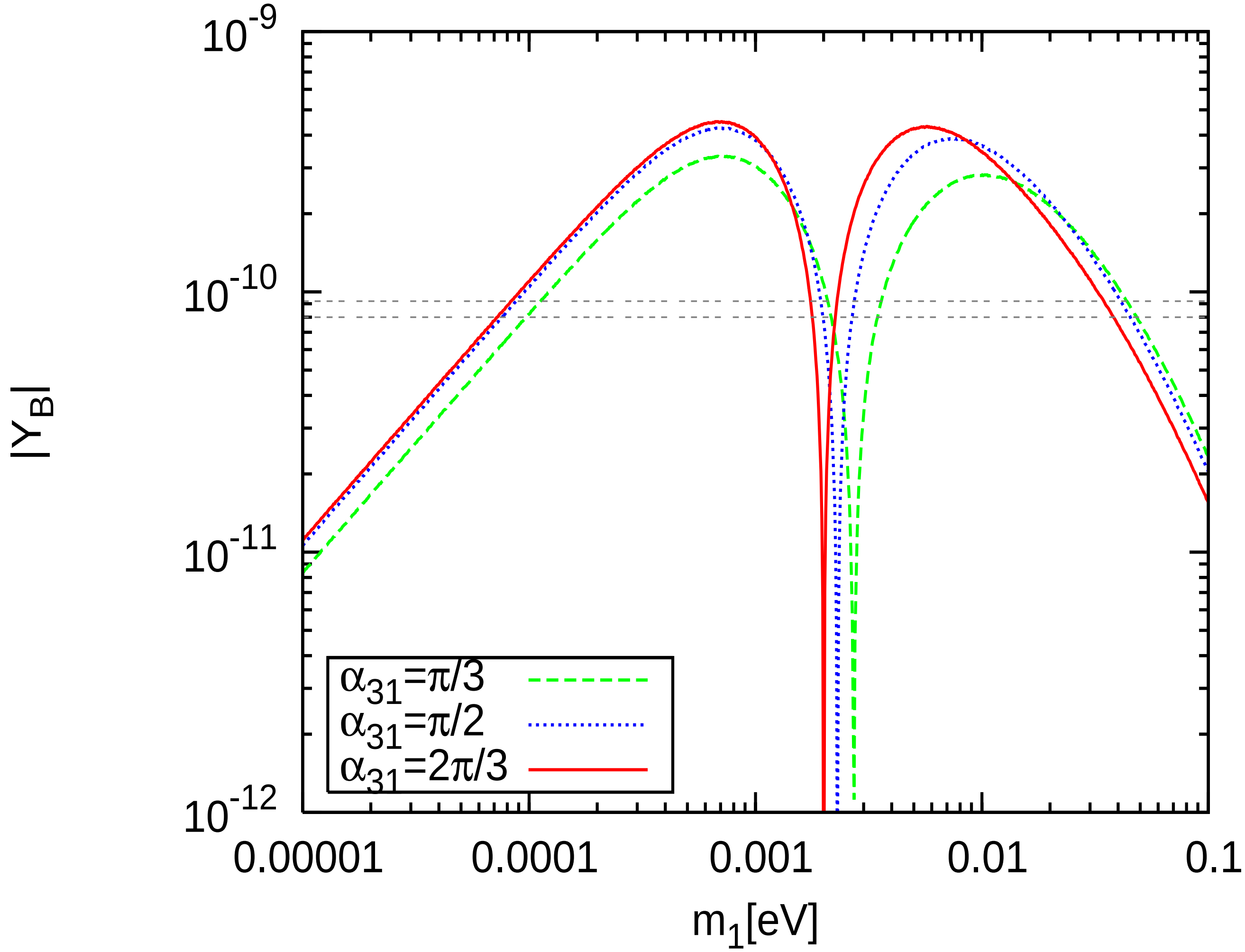
$m_3 \ll m_1 \ll m_2$ (IH), $R_{11} = 0$, real $R_{12}R_{13}$, Majorana CPV;
$\alpha_{32} = \pi/2$, $s_{13} = 0$, $M_1 = 10^{11}$ GeV; i) sgn$(R_{12}R_{13}) = +1$; ii) sgn$(R_{12}R_{13}) = -1$.

E. Molinaro, S.T.P., T. Shindou, Y. Takanishi, 2007



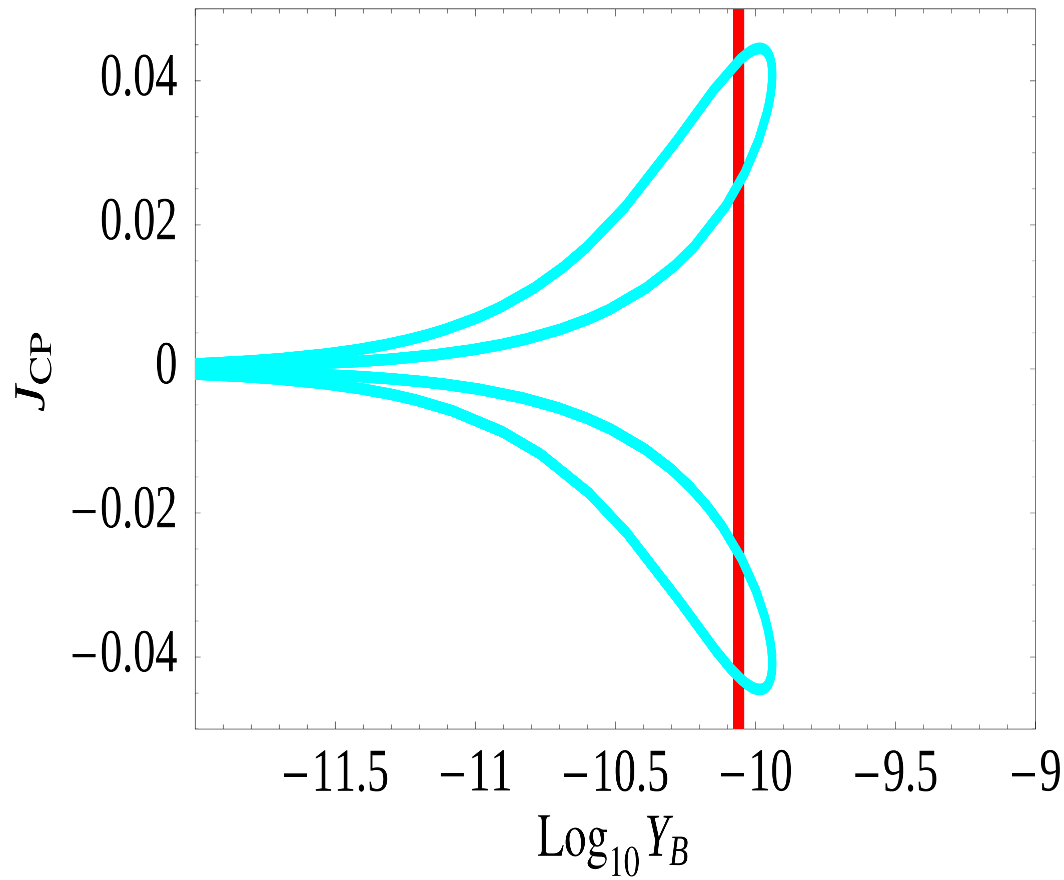
$m_3 \ll m_1 \ll m_2$ (IH), $R_{11} = 0$, real $R_{12}R_{13}$, Dirac CPV, $\alpha_{32} = 0$;
$s_{13} = 0.2$, $\delta = \pi/2$, $M_1 = 10^{11}$ GeV; i) sgn$(R_{12}R_{13}) = +1$; ii) sgn$(R_{12}R_{13}) = -1$;
i) $\sin^2\theta_{23} = 0.50; 0.35; 0.64$ (red solid, dotted, dash-dotted lines);
ii) $\sin^2\theta_{23} = 0.50$ (blue dashed line);

E. Molinaro, S.T.P., T. Shindou, Y. Takanishi, 2007



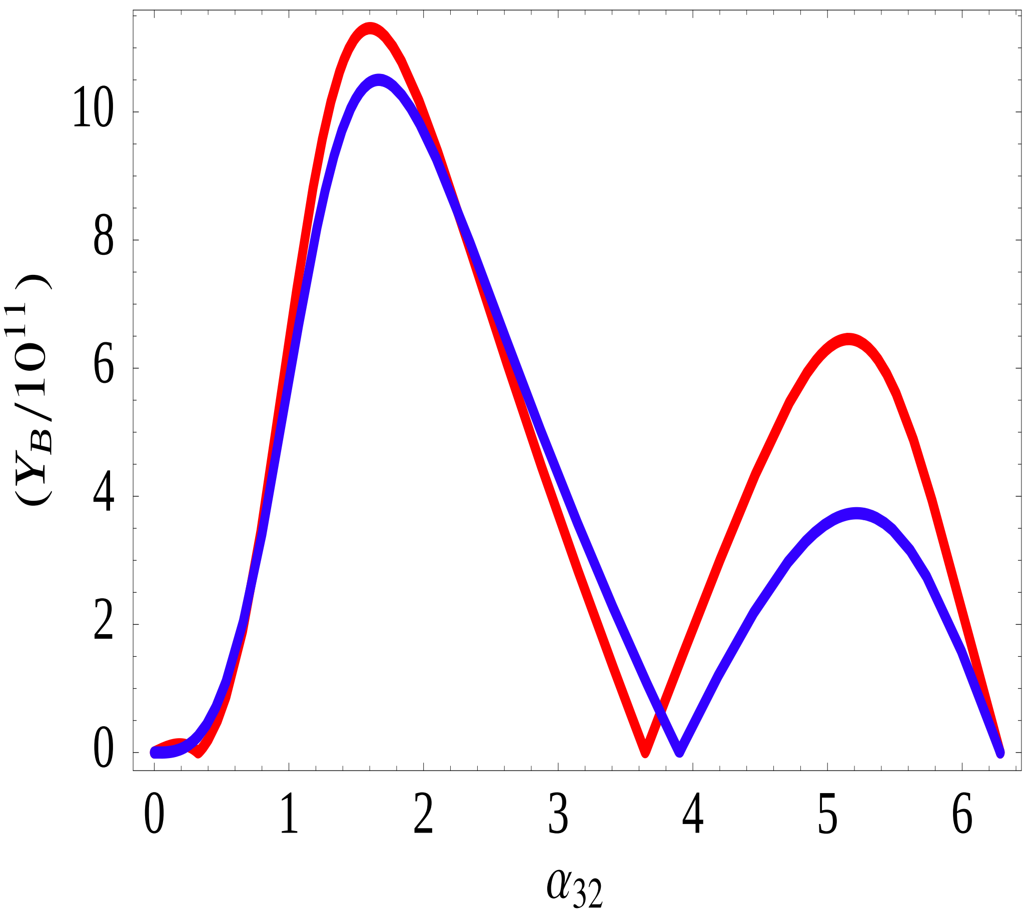
$m_1 < m_2 < m_3$ (NO(NH)), $R_{12} = 0$, real $R_{11}R_{13}$, Majorana CPV, $s_{13} = 0$;
sgn$(R_{11}R_{13}) = -1$, $\sin^2\theta_{23} = 0.50$, $M_1 = 3 \times 10^{11}$ GeV;
$\alpha_{32} = 2\pi/3; \pi/2; \pi/3$ (red, blue, green lines).

E. Molinaro, S.T.P., T. Shindou, Y. Takanishi, 2007



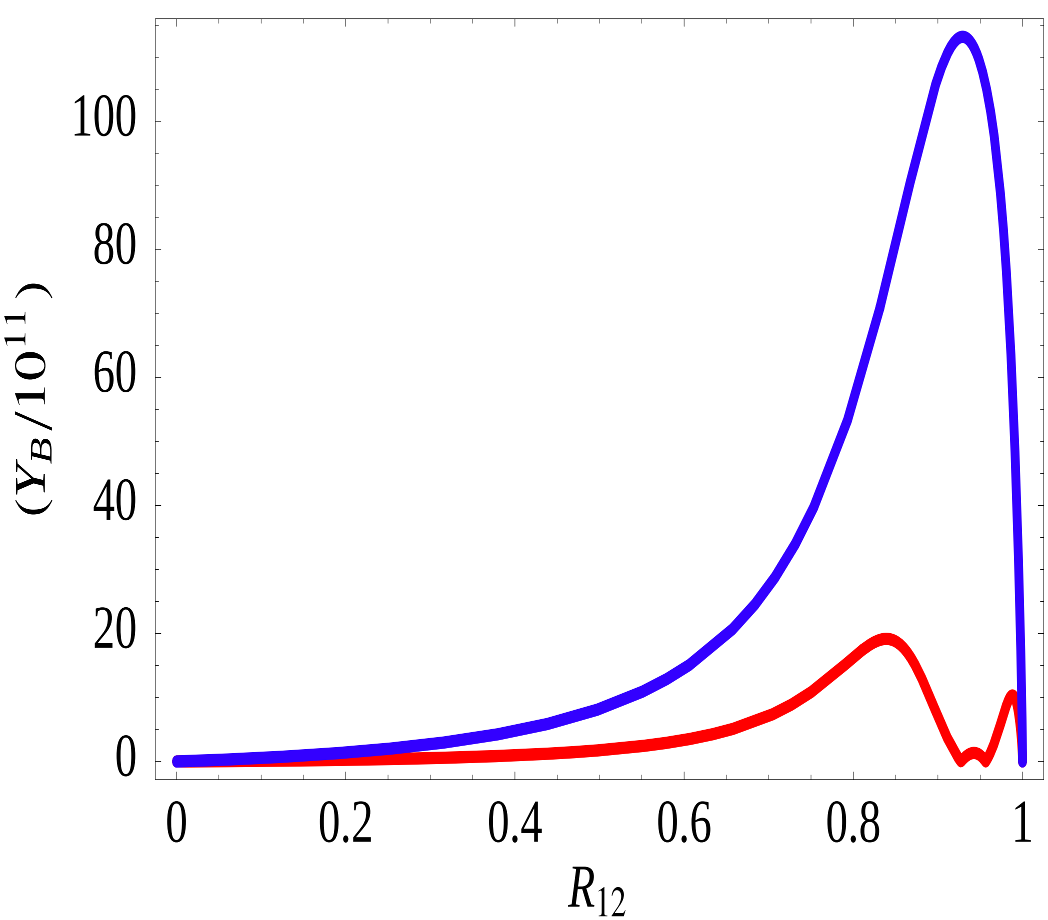
$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$; $M_1 = 5 \times 10^{11}$ GeV;
Dirac CP-violation, $\alpha_{32} = 0\ (2\pi)$;
$|R_{12}| = 0.86$, $|R_{13}| = 0.51$, sign $(R_{12}R_{13}) = +1\ (-1)$ $(\beta_{23} = 0\ (\pi)$, $\kappa' = +1)$;
The red region denotes the 2σ allowed range of Y_B.

S. Pascoli, S.T.P., A. Riotto, 2006.

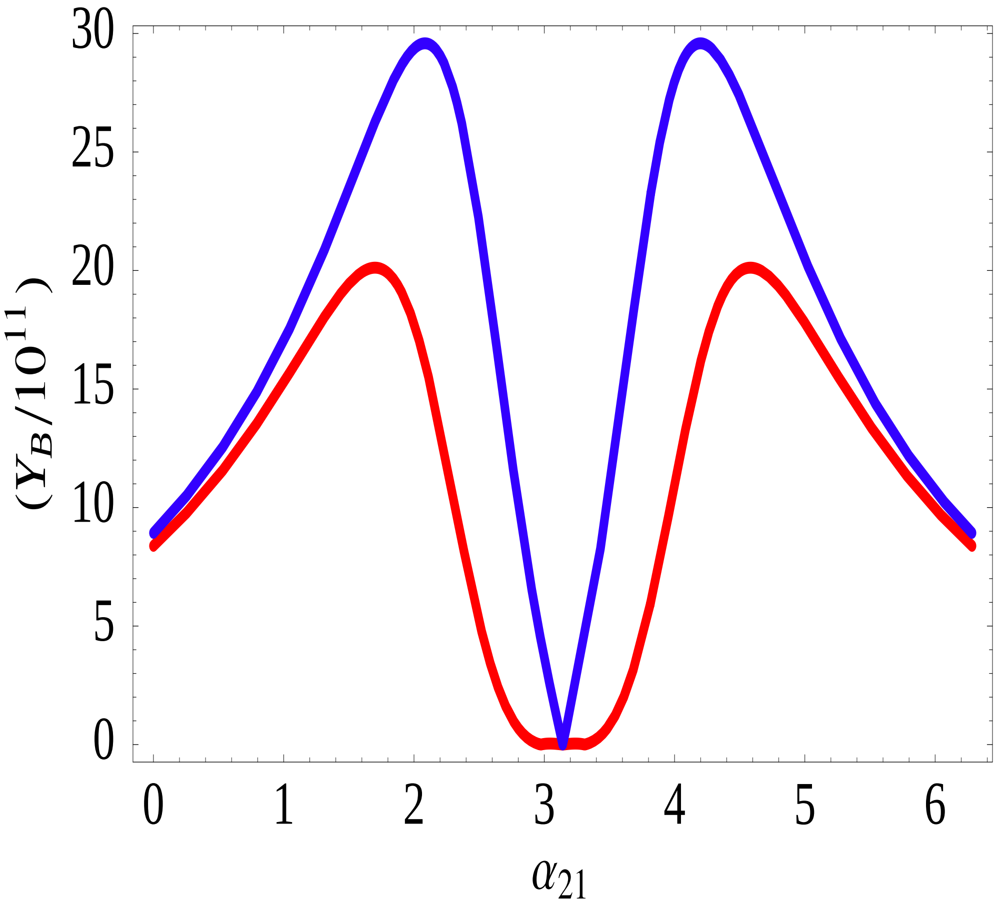


$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$; Majorana CP-violation, $\delta = 0$;
real R_{12}, R_{13}, $|R_{12}| = 0.92$, $|R_{13}| = 0.39$, $\text{sgn}(R_{12}R_{13}) = +1$ ($\beta_{23} = 0$, $\kappa = +1$);
$M_1 = 5 \times 10^{10}$ GeV, $s_{13} = 0$ (blue line) and 0.2 (red line).

S. Pascoli, S.T.P., A. Riotto, 2006.

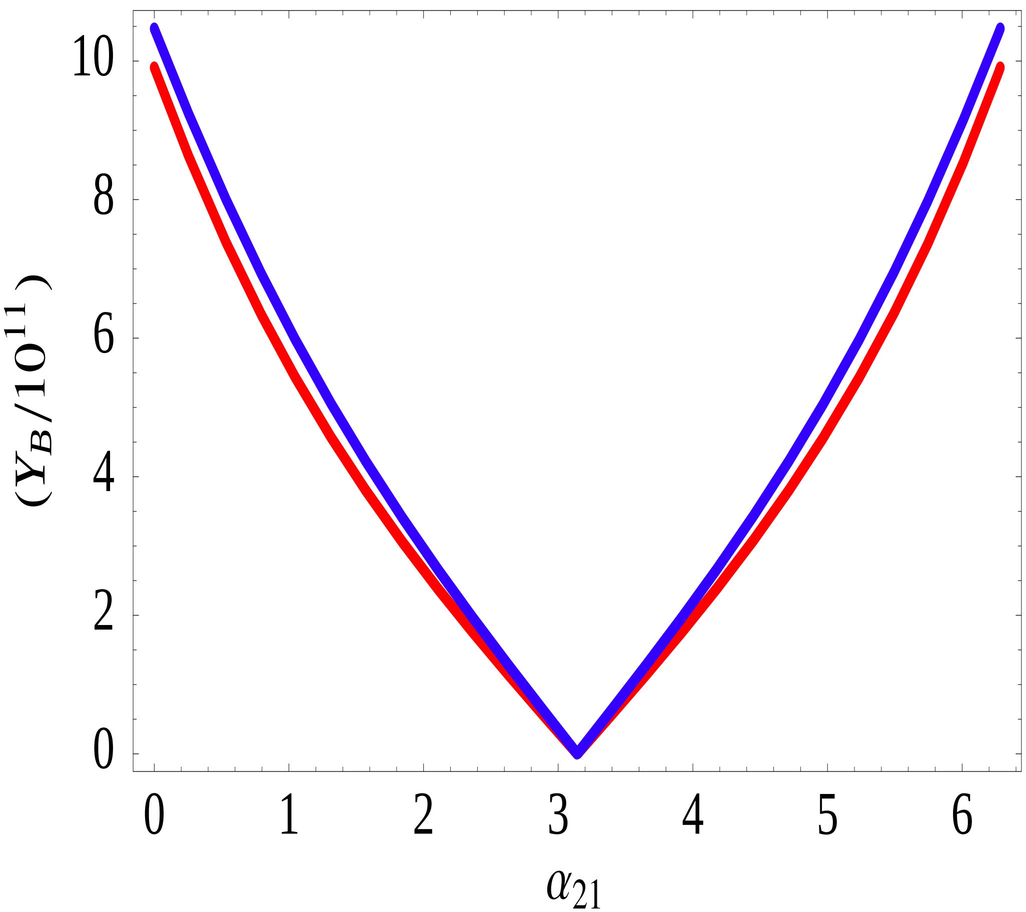


$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$; $M_1 = 5 \times 10^{11}$ GeV;
real R_{12}, R_{13}, sign$\,(R_{12}R_{13}) = +1$, $R_{12}^2 + R_{13}^2 = 1$, $s_{13} = 0.20$;
a) Majorana CP-violation (blue line), $\delta = 0$ and $\alpha_{32} = \pi/2$ ($\kappa = +1$);
b) Dirac CP-violation (red line), $\delta = \pi/2$ and $\alpha_{32} = 0$ ($\kappa' = +1$);
$\Delta m^2_{\odot}$, $\sin^2\theta_{12}$, Δm^2_{31}, $\sin^2 2\theta_{23}$ - fixed at their best fit values.



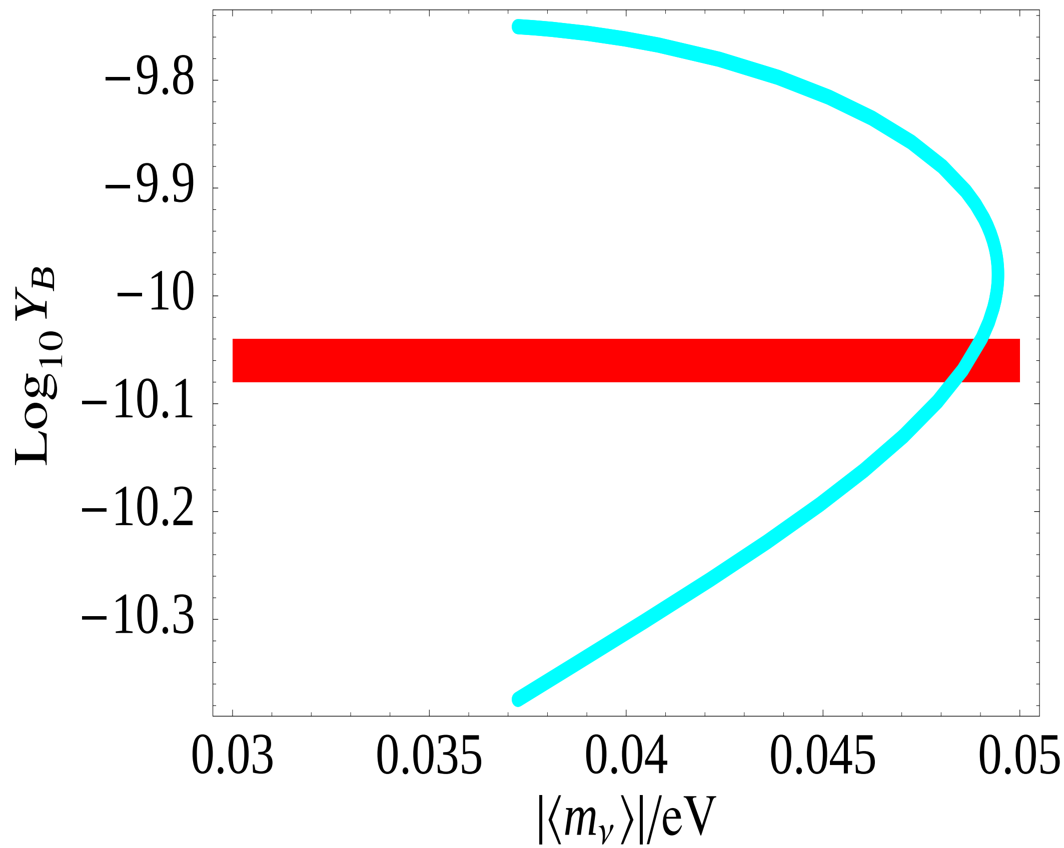
$M_1 \ll M_2 \ll M_3$, $m_3 \ll m_1 < m_2$; $M_1 = 2 \times 10^{11}$ GeV;
Majorana CP-violation, $\delta = 0$;
purely imaginary $R_{11}R_{12} = i\kappa|R_{11}R_{12}|$, $\kappa = -1$, $|R_{11}|^2 - |R_{12}|^2 = 1$, $|R_{11}| = 1.2$;
$s_{13} = 0$ (blue line) and 0.2 (red line).

S. Pascoli, S.T.P., A. Riotto, 2006.



$M_1 \ll M_2 \ll M_3$, $m_3 \ll m_1 < m_2$; $M_1 = 2 \times 10^{11}$ GeV;
Majorana CP-violation, $\delta = 0$;
purely imaginary $R_{11}R_{12} = i\kappa|R_{11}R_{12}|$, $\kappa = +1$, $|R_{11}|^2 - |R_{12}|^2 = 1$, $|R_{11}| = 1.05$;
$s_{13} = 0$ (blue line) and 0.2 (red line).

S. Pascoli, S.T.P., A. Riotto, 2006.



$M_1 \ll M_2 \ll M_3$, $m_3 \ll m_1 < m_2$; $M_1 = 2 \times 10^{11}$ GeV;
Majorana CP-violation, $\delta = 0$, $s_{13} = 0$;
purely imaginary $R_{11}R_{12} = i\kappa|R_{11}R_{12}|$, $\kappa = +1$ $|R_{11}|^2 - |R_{12}|^2 = 1$, $|R_{11}| = 1.05$.
The Majorana phase α_{21} is varied in the interval $[-\pi/2, \pi/2]$.

S. Pascoli, S.T.P., A. Riotto, 2006.

Low Energy Leptonic CPV and Leptogenesis: Summary

Leptogenesis: see-saw mechanism; N_j - heavy RH ν's;
N_j, ν_k - Majorana particles

N_j: $M_1 \ll M_2 \ll M_3$

The observed value of the baryon asymmetry of the Universe can be generated

A. CP-violation due to the Dirac phase δ in $U_{\rm PMNS}$, no other sources of CPV (Majorana phases in $U_{\rm PMNS}$ equal to 0, etc.)

$m_1 \ll m_2 \ll m_3$ (NH):

$$|\sin\theta_{13}\sin\delta| \gtrsim 0.09\,, \qquad \sin\theta_{13} \gtrsim 0.09\,; \quad |J_{\rm CP}| \gtrsim 2.0 \times 10^{-2}$$

$m_3 \ll m_1 < m_2$ (IH):

$$|\sin\theta_{13}\sin\delta| \gtrsim 0.02\,, \qquad \sin\theta_{13} \gtrsim 0.02\,; \quad |J_{\rm CP}| \gtrsim 4.6 \times 10^{-3}$$

B. CP-violation due to the Majorana phases in $U_{\rm PMNS}$, no other sources of CPV (Dirac phase in $U_{\rm PMNS}$ equal to 0, etc.)

C. CP-violation due to both Dirac and Majorana phases in $U_{\rm PMNS}$, no other source of CPV.

S. Pascoli, S.T.P., A. Riotto, 2006.

Conclusions

Determining the nature - Dirac or Majorana, of massive neutrinos is of fundamental importance for understanding the origin of neutrino masses.

The see-saw mechanism provides a link between ν-mass generation and BAU.
Majorana CPV phases in U_{PMNS}: $(\beta\beta)_{0\nu}$-decay, Y_{B}.
Any of the CPV phases in U_{PMNS} can be the leptogenesis CPV parameters.

Obtaining information on Dirac and Majorana CPV is a remarkably challenging problem.

Dirac and Majorana CPV may have the same source.

Low energy leptonic CPV can be directly related to the existence of BAU.

Understanding the status of the CP-symmetry in the lepton sector is of fundamental importance.

These results underline further the importance of the experiments aiming to measure the CHOOZ angle θ_{13} and of the experimental searches for Dirac and/or Majorana leptonic CP-violation at low energies.