SO(10) SUSY GUT in 5 D.

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Takeshi Fukuyama (Ritsumeikan University)

Based on the works in collaboration with
T. Kikuchi, N. Okada, A. Ilakovac and S. Meljanac.
References:
JHEP 0211 (2002); Phys. Rev. D 68 (2003); Int. J. Mod. Phys. A19 (2004); JHEP 0409 (2004); J. Math. Phys. 46 (2005); JHEP 0505 (2005); **Phys.Rev.D75 (2007)**
With N.Haba and N.Okada: works in preparation

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Minimal SO(10) model

(Babu-Mohapatra (93'); Fukuyama-Okada (01'))

- Two kinds of symmetric Yukawa couplings

$$W_Y = Y_{10}^{ij} \mathbf{16}_i H_{10} \mathbf{16}_j + Y_{126}^{ij} \mathbf{16}_i H_{126} \mathbf{16}_j$$

- Two Higgs fields are decomposed to

$$\begin{aligned} \mathbf{10} &\to (\mathbf{6},\mathbf{1},\mathbf{1}) + (\mathbf{1},\mathbf{2},\mathbf{2}) \\ \overline{\mathbf{126}} &\to (\mathbf{6},\mathbf{1},\mathbf{1}) + (\mathbf{15},\mathbf{2},\mathbf{2}) \\ &+ (\overline{\mathbf{10}},\mathbf{3},\mathbf{1}) + (\mathbf{10},\mathbf{1},\mathbf{3}) \end{aligned}$$

- SU(4) adjoint **15** have a basis, $\mathrm{diag}(1,1,1,-3)$so as to satisfy the traceless condition. Putting leptons into the 4th color, we get, so called, 'Georgi-Jarslkog' factor, -3 for leptons.

Mass relation

- All the mass matrices are descried by only two fundamental matrices.

$$M_u = c_{10} M_{10} + c_{126} M_{126}$$
$$M_d = M_{10} + M_{126}$$
$$M_D = c_{10} M_{10} - 3 c_{126} M_{126}$$
$$M_e = M_{10} - 3 M_{126}$$
$$M_R = c_R M_{126}$$
$$\to M_e = c_d \left(M_d + \kappa M_u \right) \quad (\text{`GUT relation'})$$

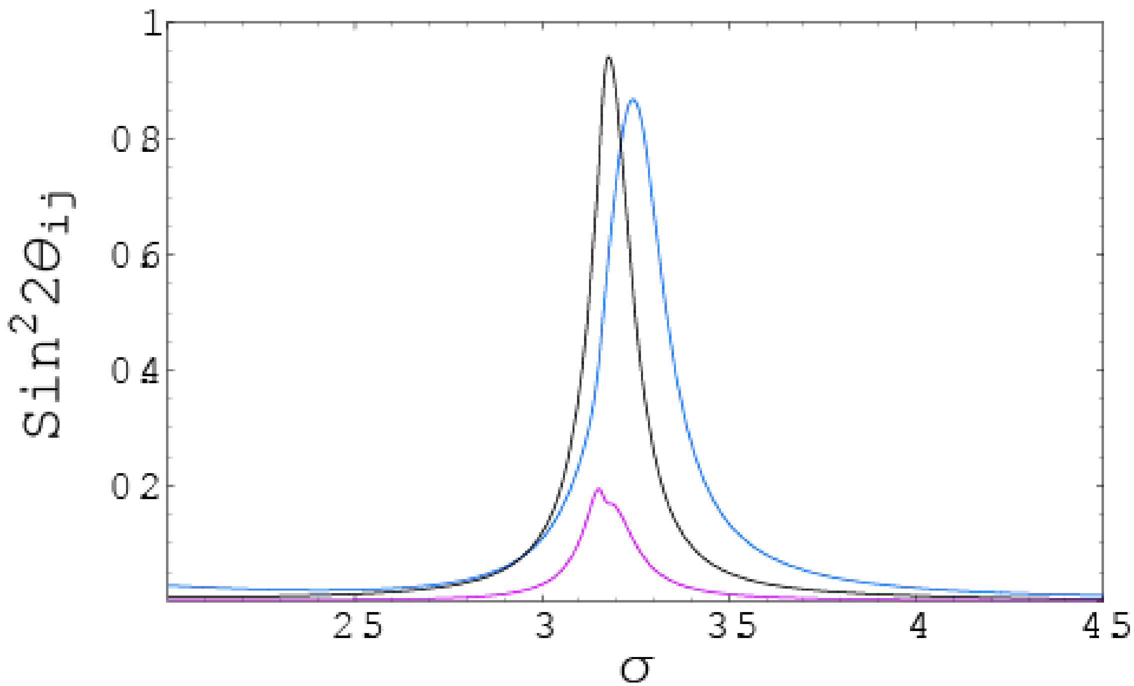
- 13 inputs : 6 quark masses, 3 angles + 1 phase in CKM matrix,
  3 charged-lepton masses.

  ⇒ fix $|c_d|$ and κ

  ⇒ **predictions in the parameters in the neutrino sector!**

Predictions in neutrino sector

- We have only one parameter $\sigma = \mathrm{arg}(c_d)$, left free. So, we can make definite predictions.



For $\sigma = 3.198$ [rad] :

$$\sin^2 2\theta_{12} \sim 0.72, \ \ \sin^2 2\theta_{23} \sim 0.90, \ \ \sin^2 2\theta_{13} \sim 0.16$$

Yukawa's are determined!

- Now, all the mass matrices have been determined!
- For example, Neutrino Dirac Yukawa coupling matrix (in the basis where charged lepton mass matrix is diagonal):

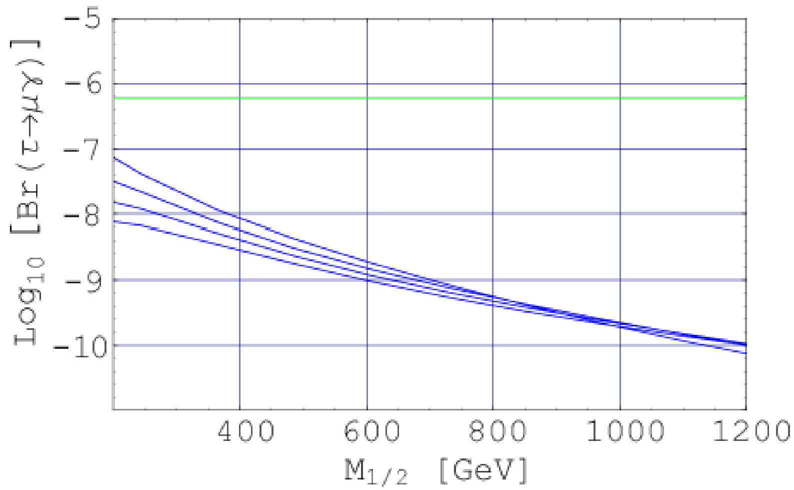
$$Y_\nu = \begin{pmatrix} -0.000135 - 0.00273i & 0.00113 + 0.0136i & 0.0339 + 0.0580i \\ 0.00759 + 0.0119i & -0.0270 - 0.00419i & -0.272 - 0.175i \\ -0.0280 + 0.00397i & 0.0635 - 0.0119i & 0.491 - 0.526i \end{pmatrix}$$

- We must check this model by proving the other phenomena related to the Yukawa couplings! (LFV, muon g-2, EDM, neutrino magnetic dipole moment (MDM), proton decay, etc.)

The prediction about $\tau \to \mu\gamma$

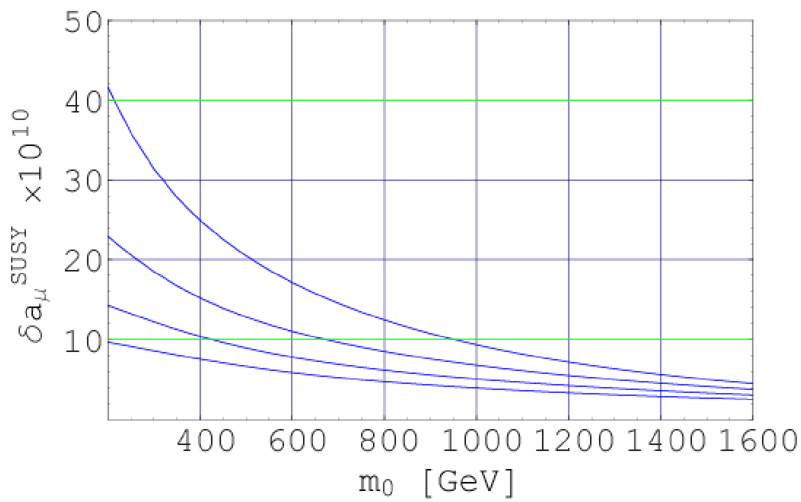
- For fixed

$$m_0 = 400,\ 600,\ 800,\ 1000\ [\mathrm{GeV}]$$



The prediction about muon g-2

- For fixed $M_{1/2} = 400,\ 600,\ 800,\ 1000\ [\mathrm{GeV}]$



- Input parameters providing $BR(\mu \to e\gamma)$ close to the present upper bound predict suitable magnitude for muon g-2.

Superpotential was fully analyzed

- Fukuyama et.al. hep-ph/0401213 v1

gave the symmetry breaking pattern

of minimal SO(10) to Standard Model,starting from

$$W = m_1\Phi^2 + m_2\Delta\overline{\Delta} + m_3H^2 + \lambda_1\Phi^3 + \lambda_2\Phi\Delta\overline{\Delta} + \lambda_3\Phi\Delta H + \lambda_4\Phi\overline{\Delta}H.$$

$$H = 10, \Delta = 126, \Phi = 210$$

2 . Problems of 4D SO(10) GUT

- Fast Proton Decay.
- Many intermediate energy scales break the gauge coupling unification since we have five directions which are singlet under G_{321}.

Proton decay in SUSY GUT

- In SUSY models, the color triplet Higgsinos mediate the proton decay induced by the following baryon and lepton number violating dimension five operator

$$W = C_L^{ijkl} Q^i Q^j Q^k L^l$$

- In the minimal SO(10) model, and the Wilson coefficients can be written as:

$$C_L^{ijkl} = (Y_{10},\ Y_{126})^{ij} \left(M_C^{-1}\right) \begin{pmatrix} Y_{10} \\ Y_{126} \end{pmatrix}^{kl}$$

M_C is the effective color triplet mass matrix. We assume their eigenvalues to be at the GUT scale $M_G \simeq 2 \times 10^{16}$ [GeV].

In the basis $\left\{H_T, \Delta_T, \overline{\Delta}_T, \Phi_T, \Delta'_{\overline{T}}\right\}$, the mass matrix reads

$$M_{\text{triplet}} \equiv \begin{pmatrix} 2m_3 & -\frac{\lambda_4\phi_1}{\sqrt{10}} - \frac{\lambda_4\phi_2}{\sqrt{30}} & -\frac{\lambda_3\phi_1}{\sqrt{10}} + \frac{\lambda_3\phi_2}{\sqrt{30}} & \frac{\lambda_4\overline{v_R}}{\sqrt{5}} & -\frac{\sqrt{2}\lambda_4\phi_3}{\sqrt{15}} \\ -\frac{\lambda_3\phi_1}{\sqrt{10}} - \frac{\lambda_3\phi_2}{\sqrt{30}} & m_2 & 0 & -\frac{\lambda_2\overline{v_R}}{10\sqrt{3}} & \frac{\lambda_2\phi_3}{15\sqrt{2}} \\ -\frac{\lambda_4\phi_1}{\sqrt{10}} + \frac{\lambda_4\phi_2}{\sqrt{30}} & 0 & m_2 & 0 & 0 \\ \frac{\lambda_3 v_R}{\sqrt{5}} & -\frac{\lambda_2 v_R}{10\sqrt{3}} & 0 & \overline{m}_{44} & -\frac{\lambda_2 v_R}{5\sqrt{6}} \\ -\frac{\sqrt{2}\lambda_3\phi_3}{\sqrt{15}} & \frac{\lambda_2\phi_3}{15\sqrt{2}} & 0 & -\frac{\lambda_2\overline{v_R}}{5\sqrt{6}} & \overline{m}_{55} \end{pmatrix}$$

where $\overline{m}_{44} \equiv 2m_1 + \frac{\lambda_1\phi_1}{\sqrt{6}} + \frac{\lambda_1\phi_2}{3\sqrt{2}} + \frac{2\lambda_1\phi_3}{3}$ and $\overline{m}_{55} \equiv m_2 + \frac{\lambda_2\phi_1}{10\sqrt{6}} + \frac{\lambda_2\phi_2}{30\sqrt{2}}$.

The corresponding mass terms of the superpotential read

$$W_m = \left(H_{\overline{T}}, \Delta_{\overline{T}}, \overline{\Delta}_{\overline{T}}, \Phi_{\overline{T}}, \Delta'_{\overline{T}}\right)\, M_{\text{triplet}}\, \left(H_T, \overline{\Delta}_T, \Delta_T, \Phi_T, \overline{\Delta}'_T\right)^{\mathsf{T}}.$$

$$
\begin{aligned}
W_Y = {} & Y_{10}^{ij}\, H_{\overline{T}} \left(q_i \ell_j + u_i^c d_j^c \right) + Y_{126}^{ij}\, \overline{\Delta}_{\overline{T}} \left(q_i \ell_j + u_i^c d_j^c \right) \\
& + Y_{10}^{ij}\, H_T \left(\frac{1}{2} q_i q_j + u_i^c e_j^c + d_i^c \nu_j^c \right) \\
& + Y_{126}^{ij}\, \overline{\Delta}_T \left(\frac{1}{2} q_i q_j + u_i^c e_j^c + d_i^c \nu_j^c \right) \\
& + Y_{126}^{ij}\, \overline{\Delta}'_T \left(u_i^c e_j^c + d_i^c \nu_j^c \right) .
\end{aligned}
\tag{6.1}
$$

Here we have defined

$$
H_{\overline{T}} \equiv H_{(6,1,1)}^{(\overline{3},1,\frac{1}{3})} , \; H_T \equiv H_{(6,1,1)}^{(3,1,-\frac{1}{3})} , \; \overline{\Delta}_{\overline{T}} \equiv \overline{\Delta}_{(6,1,1)}^{(\overline{3},1,\frac{1}{3})} , \; \overline{\Delta}_T \equiv \overline{\Delta}_{(6,1,1)}^{(3,1,-\frac{1}{3})} , \; \overline{\Delta}'_T \equiv \overline{\Delta}_{(10,1,3)}^{(3,1,-\frac{1}{3})} . \tag{6.2}
$$

Discard (6,1,1) in the PS model

Proton decay rate formula

The proton decay mode via the dimension five operator with the Wino dressing diagram is found to be dominant, and leads to the proton decay process, $p \to K^+\overline{\nu}$. The decay rate for this process is approximately estimated as

$$\begin{aligned}\Gamma(p \to K^+\overline{\nu}) &\simeq \frac{m_p}{32\pi f_\pi^2}\, |\beta_H|^2 \times |A_L A_S|^2 \times \left(\frac{\alpha_2}{4\pi}\right)^2 \frac{1}{m_S^2} \\ &\times \ |C_L^{2311} - C_L^{1312} + \lambda \left(C_L^{2312} - C_L^{1322}\right)|^2 \\ &\times \ 5.0 \times 10^{31}\,[\text{years}^{-1}/\text{GeV}]\end{aligned}$$

where $m_S = m_{\tilde{q}}^2 / M_{\widetilde{W}}$. We assume the sparticle mass spectra as $m_{\tilde{q}} = 1$ [TeV], $M_{\widetilde{W}} = 100$ [GeV], thus $m_S = 10$ [TeV].

Parameters in Higgs sector-(i)

For the <u>degenerate</u> mass spectra of the color triplet Higgsinos at the GUT scale $M_G \sim 10^{16}$ [GeV], we have 3 parameters characterizing the mass matrix for the color triplet, which we vary them freely. Then, we can parameterize the 2×2 mass matrix as

$$M_C = M_G \mathbf{1}_{2\times 2} \times U,$$

with the unitary matrix,

$$U = e^{i\varphi\sigma_3} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} e^{i\varphi'\sigma_3}$$

3 parameters $(\theta, \varphi, \varphi')$ exist in this model.

Parameters in Higgs sector-(ii)

For the non-degenerate mass spectra of the color triplet Higgsinos assuming one of them at the GUT scale $M_G \sim 10^{16}$ [GeV], we have 5+1 parameters characterizing the mass matrix for the color triplet. Then, we can parameterize the 2×2 mass matrix as

$$M_C = M_G \times U_1 \,\mathrm{diag}(1, r)\, U_2,$$

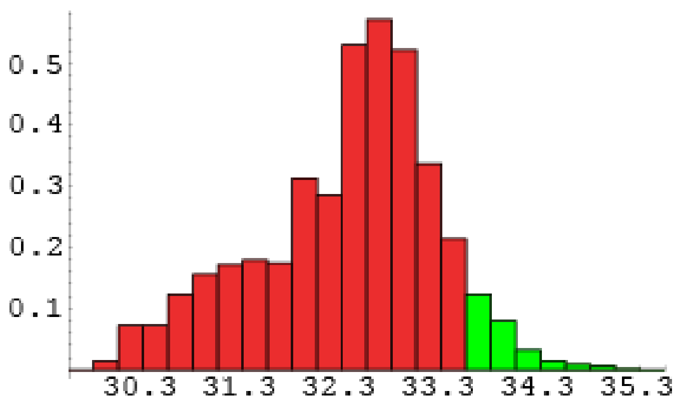
with the unitary matrices, which include 5 parameters,

$$U_1 = e^{i\varphi_1 \sigma_3} \begin{pmatrix} \cos\theta_1 & \sin\theta_1 \\ -\sin\theta_1 & \cos\theta_1 \end{pmatrix} e^{i{\varphi'}_1 \sigma_3}, U_2 = \begin{pmatrix} \cos\theta_2 & \sin\theta_2 \\ -\sin\theta_2 & \cos\theta_2 \end{pmatrix} e^{i{\varphi'}_2 \sigma_3}.$$

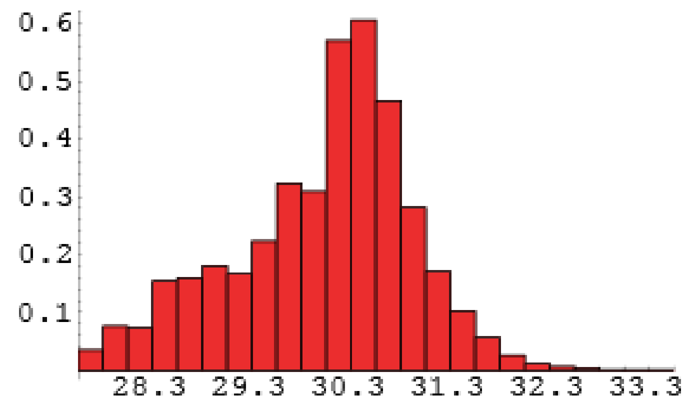
5+1 parameters $(\theta_1, \theta_2, \varphi_1, \varphi'_1, \varphi'_2; r)$, exist in this model.

Proton decay in SO(10) GUT-(i)

$\tan\beta = 2.5$



$\tan\beta = 10$



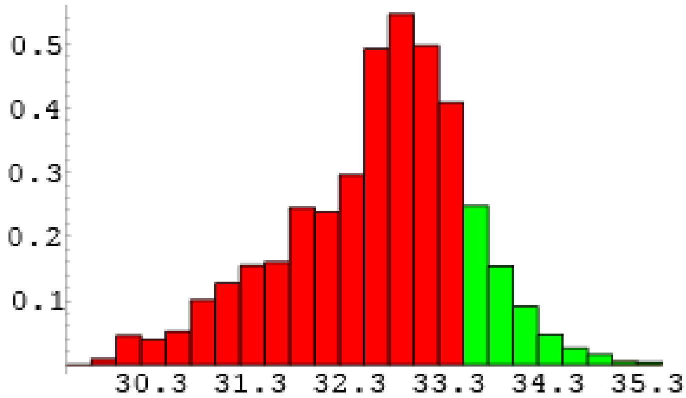
$$\tau(p \to K^+\bar{\nu}) \propto \tan^2\beta$$

The green region is the allowed region.
In case of $\tan\beta = 2.5$, there is large enough parameter space which cancels the proton decay rate, though it is very tiny region in case of $\tan\beta = 10$.

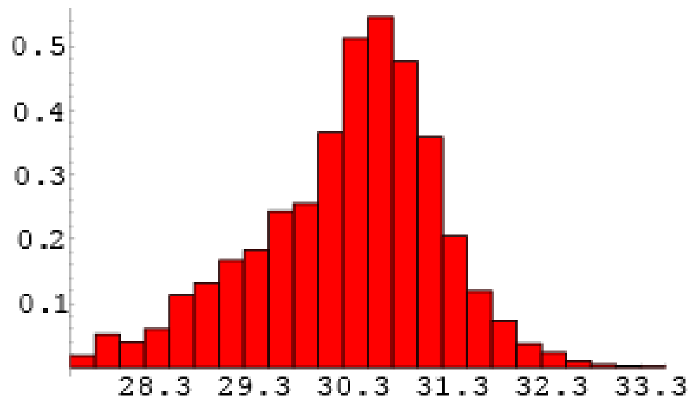
Proton decay in SO(10) GUT-(ii)

For a fixed ratio $M_{C_2}/M_{C_1} = 3$, we varied all the other parameters as possible.

$\tan\beta = 2.5$

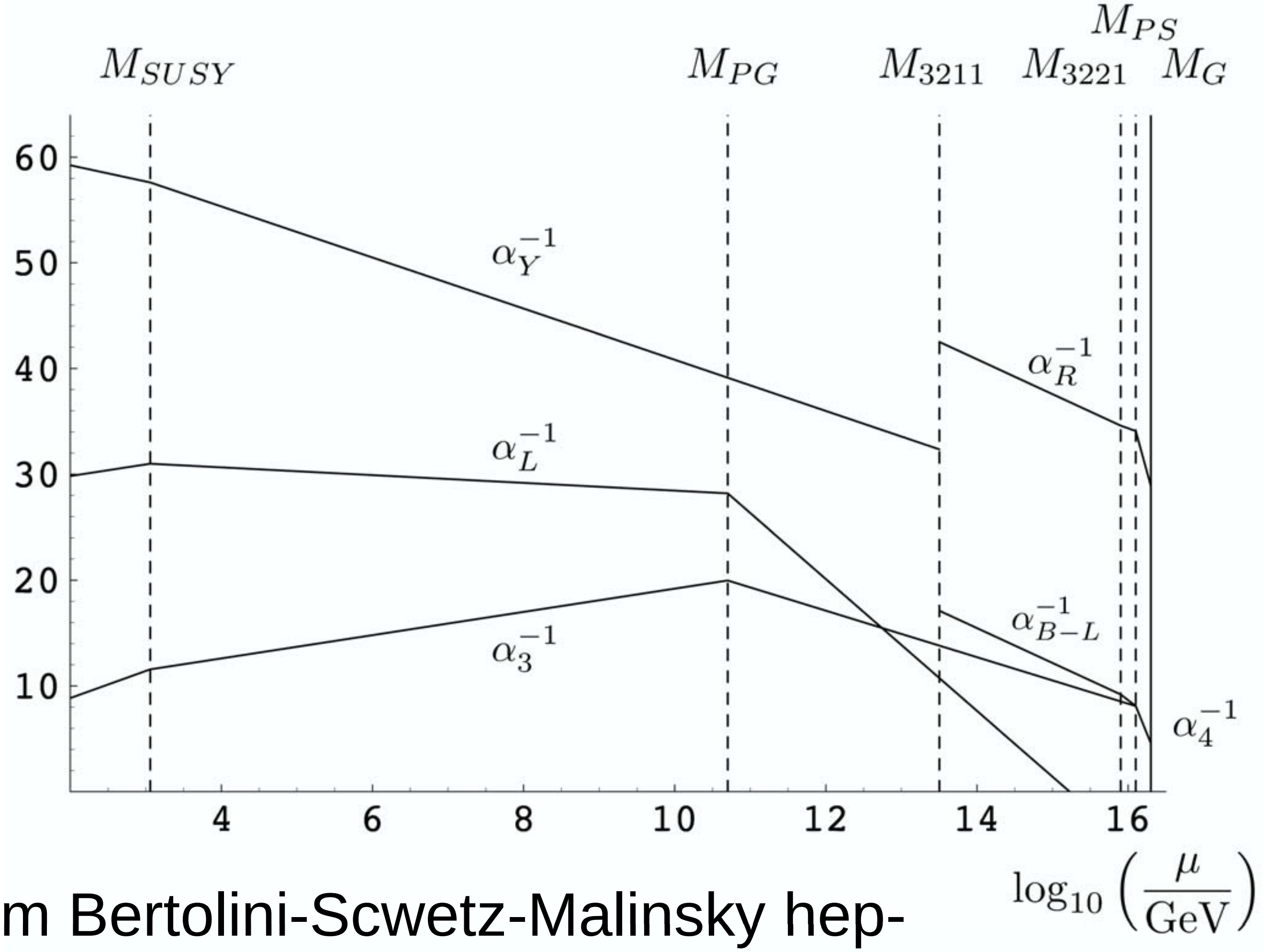


$\tan\beta = 10$



The green region is the allowed region.
In case of $\tan\beta = 2.5$, there is large enough parameter space which cancels the proton decay rate, though it is very tiny region in case of $\tan\beta = 10$.

The gauge coupling unification



From Bertolini-Scwetz-Malinsky hep-ph/0605006

Solution by warped extra dimension -First approach-

1. A variety of Higgs mass spectra destroys the successful gauge coupling unification in the MSSM. Especially, the existence of the intermediate mass scale for the right-handed neutrino cause a problem.

$$\boxed{M_R \ll M_G \;\Leftrightarrow\; \text{gauge coupling unification}}$$

2. This model has a cut off scale at the GUT scale. It means that a concrete UV completion of the model is necessary to be considered.

$$\boxed{\alpha_G \gg 1 \;\Leftrightarrow\; \Lambda_{UV} = M_G \text{ ; UV completion}}$$

We explore to solve these problems by changing the 4D flat space to the 5D Randall-Sundrum type warped background.

1. This model can easily provide a natural suppression for the Yukawa couplings by a wave function localization.
2. UV completion is provided by a strong gravity. (cf. AdS/CFT)

In order to realize the gauge coupling unification, the simplest way is to put all the VEV's at the GUT scale.

On the other hand, neutrino oscillation data shows the existence of the intermediate mass scale, which may destabilize the successful gauge coupling unification in the MSSM.

$$M_\nu = M_D^T M_R^{-1} M_D \Rightarrow M_R \sim 10^{11-14} GeV$$

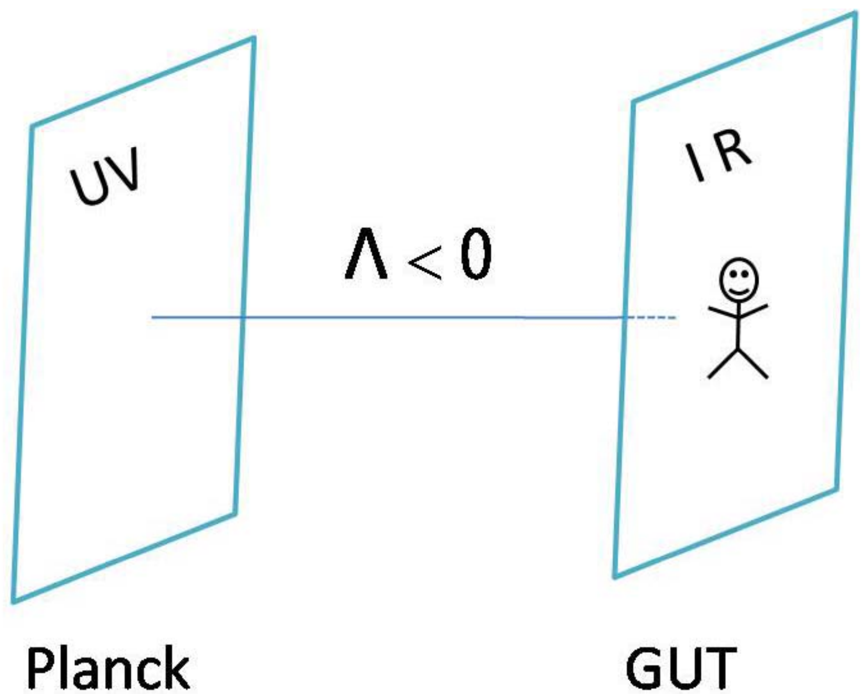
How to solve this mild hierarchy problem?

$$M_R \ll M_G$$

It is necessary to have an additional suppression as follows

$$M_R = c M_G, \;\; c = \mathcal{O}(10^{-2} - 10^{-5})$$

World of Extra Dimension



A solution to the mass hierarchy problem

In these days, there is a natural solution to provide a large mass hierarchy. In the Randall-Sundrum scenario, the exponentially warped metric can be used to explain any mass hierarchy.

$$c = \mathcal{O}(10^{-2} - 10^{-5}) = e^{-kr_c\pi} \Rightarrow kr_c = 1.5 - 3.7$$

The Setup

We assume a 5D warped space:

$$ds^2 = e^{-2kr_c|y|}\eta_{\mu\nu}dx^\mu dx^\nu + r_c^2 dy^2$$

In SUSY models in 5D, any chiral multiplets become a part of hypermultiplet and vector multiplet is extended to include an adjoint scalar.

$$\mathbb{H} = (H, H^c), \; \mathbb{V} = (v_\mu, \lambda, \bar{\lambda}, \phi)$$

By assigning an *even/odd* parity for H/H^c, only H has a zero mode wave function. This assignment also allows a bulk mass term for them.

Effect of the VEV for the adjoint scalar

If we take into account of the VEV for the adjoint scalar in the vector multiplet, the bulk mass term is shifted as follows [Kitano-Li (03')] :

$$\mathcal{S} = \int d^5x \int d^2\theta e^{-3kr_c|y|} H^c \left[\partial_5 - \left(\frac{3}{2} - c\right) kr_c \epsilon(y) + g\left\langle\phi\right\rangle\right] H$$
$$+h.c.$$

For the parity conservation, the adjoint VEV which has an odd parity should have the following form:

$$\langle\phi\rangle = v^{3/2}\epsilon(y)$$

In the context of the Left-Right symmetric gauge theory, providing the VEV only for the right-handed part provides an explanation for the mild hierarchy.

$$M_R/M_G = e^{-g\langle\phi\rangle\pi}, \quad \langle\phi\rangle \propto SU(2)_R,$$
$$g\left\langle\phi\right\rangle = 1.5 - 3.7$$

$$\mathbf{45} = \mathbf{1}_0 \oplus \mathbf{10}_{+4} \oplus \overline{\mathbf{10}}_{-4} \oplus \mathbf{24}_0 \ .$$

The $\mathbf{10}$ Higgs multiplet and the $\overline{\mathbf{126}}$ Higgs multiplet are decomposed under $\mathrm{SU}(5) \times \mathrm{U}(1)_X$ as

$$\begin{aligned} \mathbf{10} &= \mathbf{5}_{+2} \oplus \overline{\mathbf{5}}_{-2} \ , \\ \overline{\mathbf{126}} &= \mathbf{1}_{+10} \oplus \mathbf{5}_{+2} \oplus \overline{\mathbf{10}}_{+6} \oplus \mathbf{15}_{-6} \oplus \overline{\mathbf{45}}_{-2} \oplus \mathbf{50}_{+2} \ . \end{aligned}$$

In this decomposition, the coupling between a bulk Higgs multiplet and the $\mathrm{U}(1)_X$ component in χ is proportional to $\mathrm{U}(1)_X$ charge,

$$\mathcal{L}_{int} \supset \frac{1}{2} \int d^2\theta \omega^3 Q_X \langle \Sigma_X \rangle H^c H + h.c. \ , \tag{19}$$

and thus each component effectively obtains the different bulk mass term,

$$\left(\frac{3}{2} - c \right) k r_c + \frac{1}{2} Q_X \langle \Sigma_X \rangle , \tag{20}$$

So we can determine the profile of wave function from group propery.

Solution by orbifold
-Second Approah-

- Kawamura('01),Hall-Nomura('01),Dermisek-Mafi('01)

Raby-H.D.Kim

$$S^1/(Z_2 \times Z_2{}^{'})$$

Proton decay is suppressed by BC.

BC breaks SU(5) into MSSM with direct no coupling with triplet, but does SO(10) into Pati-Salam which includes it in general,

- For SO(10), there are many set ups even in $S^1/(Z_2 \times Z_2^{'})$.
- y=0 visible and

  vev of PS singlet at y=pi/2 brane destroys SUSY.
- y=pi/2 visible and vev of SO(10) singlet at y=0 branes destroys SUSY – today's talk.
- In both cases, PS invariance play the crucial role for the protect from the fast proton decay.
- We can exclude (6,1,1) which was harmful, included in full SO(10) invariant theory.

Our Setup

SO(10) inv PS

inv

vev of (10,1,3)

Standard

y = 0 y =

pi/2

Inviible visible

	bulk	brane $y=\pi R/2$
Matter fields	No matter	$\psi_i = F_{Li} \oplus F_{Ri} \quad (i=1,2,3)$
Higgs sector	$H_{10},\ H'_{10}$	$(\mathbf{15},\mathbf{1},\mathbf{1})$, $(\mathbf{10},\mathbf{1},\mathbf{3})$, $(\overline{\mathbf{10}},\mathbf{1},\overline{\mathbf{3}})$

Table 2: The contents of matters and Higgs with Z parity even. $F_{L.Ri}$ are $(\mathbf{4},\mathbf{2},\mathbf{2})$ of i'th generation and given in Eq.(3)

$$\begin{pmatrix} u_r & u_y & u_b & \nu_e \\ d_r & d_y & d_b & e \end{pmatrix}_{L(R)} \equiv F_{L(R)1},$$

$$(\mathbf{4},\mathbf{2},\mathbf{1}) \times (\overline{\mathbf{4}},\mathbf{1},\mathbf{2}) = (\mathbf{15},\mathbf{2},\mathbf{2}) + (\mathbf{1},\mathbf{2},\mathbf{2}),$$

From $\tau - b$ unification at GUT scale the third generation is described by H_{10}. The deviation of the first and second generation is complimented by the $(\mathbf{15},\mathbf{2},\mathbf{2})_H$. This term is constructed from the product of $H'(\mathbf{1},\mathbf{2},\mathbf{2})$ in the bulk and $(\mathbf{15},\mathbf{1},\mathbf{1})$ in the PS brane as $H'(\mathbf{1},\mathbf{2},\mathbf{2}) \times (\mathbf{15},\mathbf{1},\mathbf{1})/M_5$, and

(P,P')	field	mass
$(+,+)$	$V(15,1,1)$, V(1,3,1),V(1,1,3),$H(1,2,2)$	$\frac{2n}{R}$
$(+,-)$	$V(6,2,2)$, $H(6,1,1)$	$\frac{(2n+1)}{R}$
$(-,+)$	$\Phi(6,2,2)$, $\hat{H}(6,1,1)$	$\frac{(2n+1)}{R}$
$(-,-)$	$\Phi(15,1,1)$, $\Phi(1,3,1)$, $\Phi(1,1,3)$, $\hat{H}(1,2,2)$	$\frac{(2n+2)}{R}$

Table 1: P and P' assignment and masses ($n \geq 0$) of fields in the vector multipet (V, Φ) and hypermultiplets (H, $\hat{H}$) under the PS group. P' even V contains the PS gauge bosons; P' odd V contains the SO(10)/PS gauge bosons.

$$\begin{pmatrix} u_r & u_y & u_b & \nu_e \\ d_r & d_y & d_b & e \end{pmatrix}_{L(R)} \equiv F_{L(R)1}, \tag{3}$$

$F_{L(R)2}$ and $F_{L(R)3}$ are likewise defined for the 2nd and 3rd generations. Note that their transformation properties are $F_{Li} = (\mathbf{4}, \mathbf{2}, \mathbf{1})$ and $F_{Ri} = (\mathbf{4}, \mathbf{1}, \mathbf{2})$ and that $(F_{Li} + \overline{F_{Ri}}\ )$ yields the **16** of SO(10). Since $(\mathbf{4}, \mathbf{2}, \mathbf{1}) \times (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2}) = (\mathbf{15}, \mathbf{2}, \mathbf{2}) + (\mathbf{1}, \mathbf{2}, \mathbf{2})$, the Dirac masses for quarks and leptons are generated by $(\mathbf{15}, \mathbf{2}, \mathbf{2})_H + (\mathbf{1}, \mathbf{2}, \mathbf{2})_H$.

From $\tau - b$ unification at GUT scale the third generation is described by H_{10}. The deviation of the first and second generation is complimented by the $(\mathbf{15}, \mathbf{2}, \mathbf{2})_H$. This term is constructed from the product of $H'(\mathbf{1}, \mathbf{2}, \mathbf{2})$ in the bulk and (**15**,**1**,**1**) in the PS brane as $H'(\mathbf{1}, \mathbf{2}, \mathbf{2}) \times (\mathbf{15}, \mathbf{1}, \mathbf{1})/M_5$, and

$$\begin{aligned}
M_u &= c_{10} M_{1,2,2} + c_{15} M_{15,2,2}\ , \\
M_d &= M_{1,2,2} + M_{15,2,2}\ , \\
M_D &= c_{10} M_{1,2,2} - 3 c_{15} M_{15,2,2}\ , \\
M_e &= M_{1,2,2} - 3 M_{15.2,2}\ , \\
M_L &= c_L M_{10,3,1}\ , \\
M_R &= c_R M_{10,1,3}\ ,
\end{aligned}$$

Gauge coupling unification

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i(M)} + \frac{1}{2\pi} b_i \ln\left(\frac{M}{\mu}\right). \quad (i = 3, 2.1) \tag{9}$$

In the PS stage $\mu > M_c$, the threshold corrections Δ_i due to KK mode in the bulk are added. b_i at G_{321} are

$$b_3 = -7,\ b_2 = -19/6,\ b_1 = 41/10 \tag{10}$$

at $M_{SUSY} > \mu > M = M_Z$ and

$$b_3 = -3,\ b_2 = 1,\ b_1 = 33/5 \tag{11}$$

at $M_c > \mu > M = M_{SUSY}$ The PS symmetry is recovered at $\mu = M_c = 1/(\pi R)$ and the matching condition holds

$$\begin{aligned} \alpha_3^{-1}(M_c) &= \alpha_4^{-1}(M_c) \\ \alpha_2^{-1}(M_c) &= \alpha_{2L}^{-1}(M_c) \qquad (12) \\ \alpha_1{-}1(M_c) &= [2\alpha_4^{-1}(M_c) + 3\alpha_{2R}^{-1}]/5 \qquad (13) \end{aligned}$$

at M_c. In the PS stage $\mu > M_c$, the threshold corrections Δ_i due to KK mode in the bulk are added,

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i(M_c)} + \frac{1}{2\pi} b_i \ln\left(\frac{M_c}{\mu}\right) + \Delta_i. \quad (i = 4, 2_L, 2_R) \tag{14}$$

The beta functions of the PS gauge coupling constants are

$$b_4 = 5,\ b_{2L} = b_{2R} = 9 \tag{15}$$

μ problem

At $y = 0$ brane, SO(10) remaind invariant but supersymmetry is broken by the F-term of SO(10) singlet S, $F\theta^2$. The interaction between S and bulk fields is

$$\begin{aligned} L &= \delta(y)[\int d^2\theta S W_a W_a / M_* + \int d^4\theta (S^+(H_{10}H_{10} + H_{10}H'_{10} + 10'_H 10'_H)/M_*) \\ &+ SS^\dagger (H_{10}H_{10} + H_{10}H'_{10} + 10'_H 10'_H)/M_*^2) + h.c.] \end{aligned} \tag{19}$$

and

$$L_\mu = \delta(y) \int d^4\theta S^\dagger (H_{10}H_{10} + H_{10}H'_{10} + H'_{10}H'_{10})]/M_* \tag{20}$$

If we select $\sqrt{F} = O(10^{10}\text{TeV})$, the first, second in Eq.(19) and Eq.(20) correctly give gaugino masses $M_{1/2}$, $B\mu$ term and μ term, respectively [20], and

$$M_{1/2} = \mu = F/M_*, \ B\mu = F^2/M_*^2. \tag{21}$$

where M_* is the gauge unification scale, $O(10^{16}$ GeV]).

Summary

4D

- The explicit construction of Higgs superpotetial revealed the precise spectra of intermediate enrgy specta to standard.
- It led to the destruction of gauge coupling unification.
- It may lead to the incompatibility with proton stability
- In order to circumvent these pathologies, we are forced to put it in extra dimensions.

5D

- We considered SO(10) in 5 D in two ways.
- One is to break SO(10) by the vev of bulk, which may explain tha mass spectra.
- Another is to break SO(10) by using both the boundary conditions and vev.

  This preserves the advantageous points of SO(10) (mass relations of Dirac fermions and saves the disadvantageous points of SO(10) (Proton decay and gauge unification) .